DES CARTES. This famous French philoſopher and mathematician wrote when he was very young, and indeed while he was engaged in the profeſſion of a ſoldier and lay in garriſon at Breda, a treatiſe called Muſicæ Compendium; which was publiſhed in the year 1617; and comprehends the following articles. 'De numero vel tempore in ſonis obſervando. De ſonorum diverſitate circa acutum et grave. De conſonantiis. De octavâ. De quintâ. De quartâ. De ditono, tertiâ minore, et ſextis. De gradibus ſive tonis muſicis. De diſſonantiis. De ratione componendi et modis. De modis.'

This Compendium of Muſic, though it is comprized in fifty-eight ſmall quarto pages, contains a great number of very curious particulars relative to the ſcience. The obſervations of Des Cartes on the effects of various meaſures are new and judicious.

'We ſay in the generall that a ſlow meaſure doth excite in us gentle and ſluggiſh motions, ſuch as a kind of languor, ſadneſſe, fear, pride, and other heavy and dull paſſions: and a more nimble and ſwift meaſure doth proportionably excite more nimble and ſprightly paſſions; ſuch as joy, anger, courage, &c. the ſame may alſo be ſayd of the double kind of percuſſion, viz. that a quadrate, or ſuch as is perpetually reſolved into equals, is ſlower and duller than a tertiate, or ſuch as doth conſiſt of three equal parts. The reaſon whereof is, becauſe this doth more poſſeſſe and imploy the ſenſe, inaſmuch as therein are more, namely 3, members to be adverted, while in the other are only 2.'

In his enumeration of the conſonances, contrary to the ſenſe of all other writers, from John De Muris to Merſennus, he excludes the uniſon; and for this very good reaſon, that 'therein is no difference of ſounds as to acute and grave; it bearing the ſame relation to conſonances, as unity doth to numbers.'

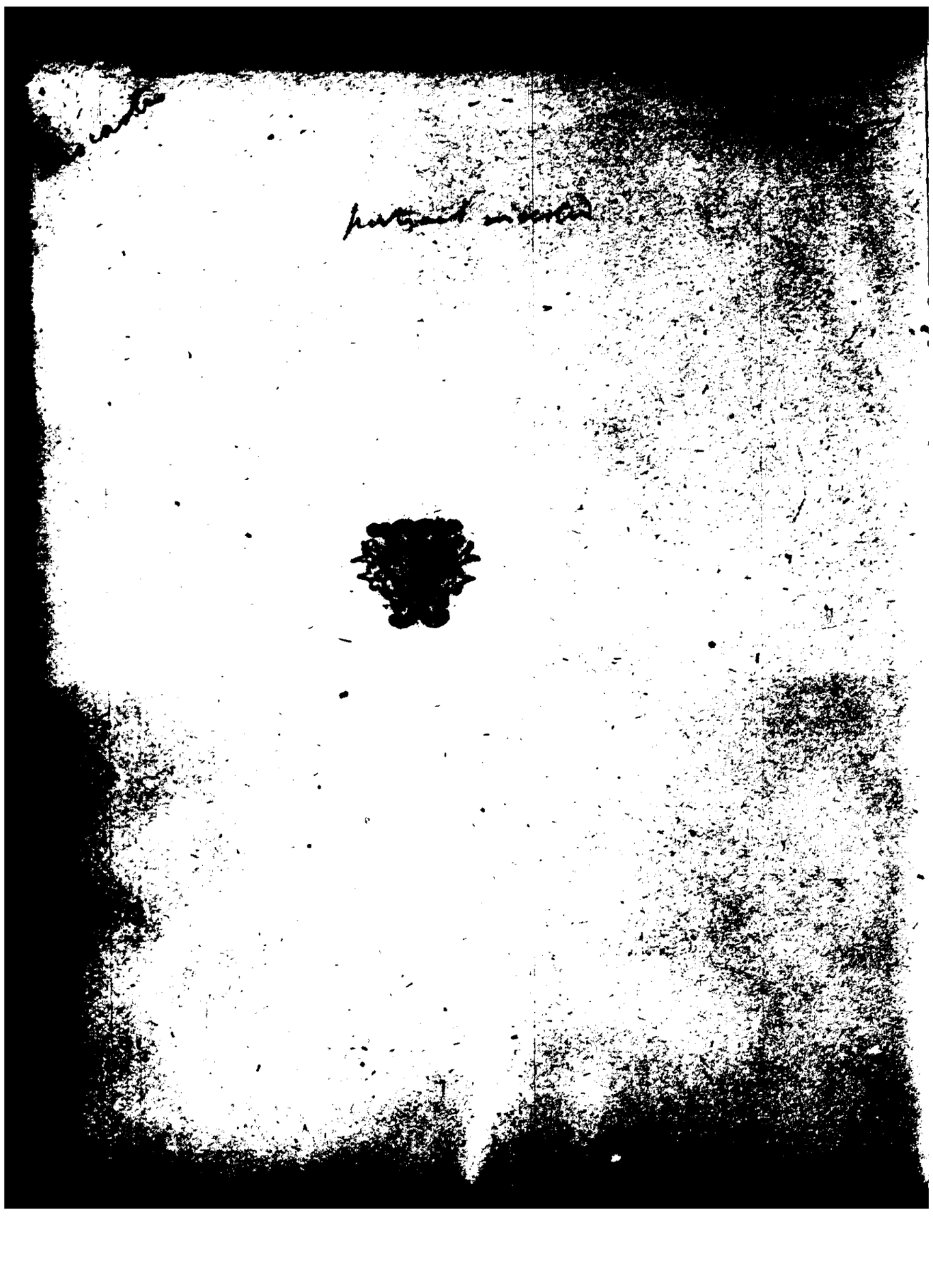
Of the two methods by which the diapaſon or octave is divided, the arithmetical and geometrical, Des Cartes, for the reaſons contained in the ſixth of his Prænotanda, prefers the former; and, for the purpoſe of adjuſting the conſonances, he propoſes the diviſion of a chord, firſt into two equal parts, and afterwards into ſmaller proportions, according to the following plan.

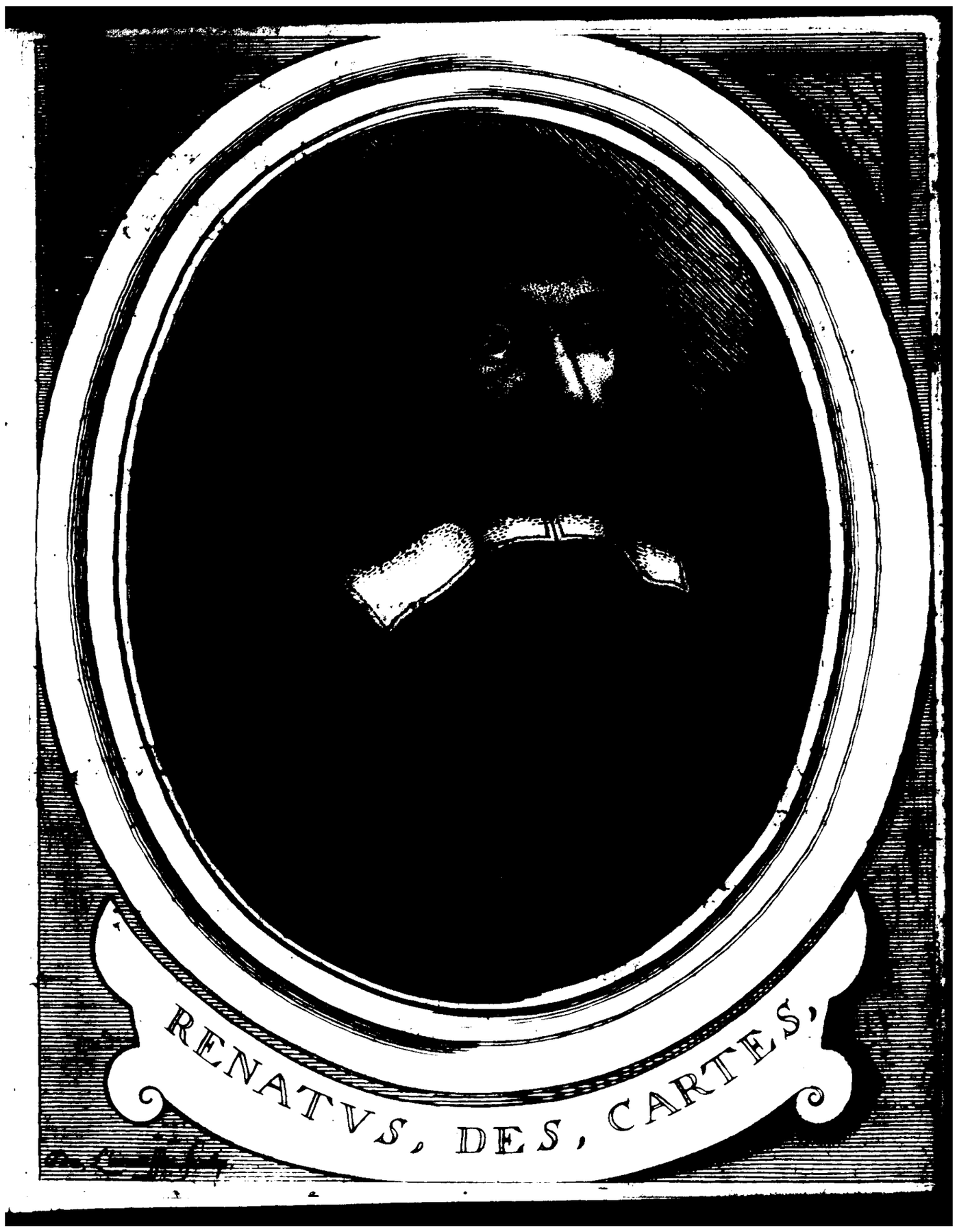
1/2	Eighth								
1/3	Twelfth	2/3	Fifth						
1/4	Fifteenth	2/4	Eighth	3/4	Fourth				
1/5	Seventeenth	2/5	Tenth Major	3/5	Sixth Major	4/5	Ditone		
1/6	Nineteenth	2/6	Twelfth	3/6	Eighth	4/6	Fifth	5/6	Third Minor

The advantages reſulting from the geometrical diviſion appear in the Syſtema Participato, mentioned by Bontempi, which conſiſted in the diviſion of the diapaſon or octave into twelve equal ſemitones by eleven mean proportionals; Des Cartes, however, rejects this diviſion, though his reaſons for ſo doing are very far from ſatisfactory.

This book was tranſlated into Engliſh in 1653, and publiſhed by a Perſon of Honour, (viz. William Lord Brouncker, preſident of the Royal Society, and the firſt appointed to that office;) with ſuch animadverſions as plainly ſhew that his lordſhip was deeply ſkilled in the theory of the ſcience; but, though he almoſt everywhere agrees with his author, he ſcruples not to aſſert, that the geometrical is preferable to the arithmetical diviſion; and, as it ſeems, with a view to a farther improvement of the Syſtema Participato, he propoſes a diviſion of the diapaſon by ſixteen mean proportionals into ſeventeen equal ſemitones; the method of which diviſion he exhibits in an algebraic proceſs, as well as in logarithms. Notwithſtanding the undoubted merit of this Compendium of Deſcartes, it contains ſome unaccountable ſingularities; of which the following extract may ſerve as a ſpecimen.

'This only thing ſeems to render the voice of man the moſt grateful of all other ſounds, that it holds the greateſt conformity to our ſpirits. Thus alſo is the voice of a friend more grateful than that of an enemy, from a ſympathy and diſpathy of affections: by the ſame reaſon perhaps that it is conceived that a drum headed with a ſheep's ſkin yields no ſound though ſtrucken, if another drum headed with a wolf's ſkin be beaten upon in the ſame room.'




RENATVS, DES, CARTES,

SONNET.

Pour un Eſprit ſi pur la Terre eſtoit impure ;
Cét Homme tout celeſte eſt monté dans les Cieux ;
Il y void clairement ces flambeaux radieux,
Dont nous n'avons icy qu'une lumiere obſcure.

De ces voutes d'azur la noble Architecture
Ravit également ſon eſprit & ſes yeux ;
Et l'élevant plus haut que ſa propre nature,
Luy fait connoiſtre enfin la nature des Dieux.

Il me ſemble deſia qu'au travers de ce verre,
Dont ſon art approchoit le Ciel, l'Onde, & la Terre,
Je le vois éclater au front du Firmament :

Et ſi l'on ſe transforme en la choſe qu'on aime ;
Comme il fut amoureux des Aſtres ſeulement,
Que le grand GALILE'E *eſt un Aſtre luy-meſme.*

COLLETET.

RENATUS DES-CARTES

EXCELLENT

COMPENDIUM OF MUSICK:

WITH

Necessary and Judicious

ANIMADVERSIONS

Thereupon.

By a Person of HONOUR.

London, Printed by *Thomas Harper*, for *Humphrey Moseley*, and are to bee sold at his Shop at the Signe of the *Princes Armes* in S. *Pauls* Church-Yard, and by *Thomas Heath* in *Coven Garden*. 1653. 2·6x

[redacted]

THE

STATIONER

To the Ingenious

READER,

&c.

SIR:

NO sooner can your Eye have taken in the Title *of this thin* Volume, *which I have, in some latitude of Assistance, Midwiv'd into this our* English *World; but you shall most willingly*

ly confesse it to be as well a sufficient Justification *to my* Industry *and* Coſt, *as a full* Elogie *to* it ſelfe: *The AUTHOR thereof, being one of the faireſt Flowers in that* Garland *of the* Mathematicks, *wherewith this* Century *being meritoriouſly adorned, may, without breach of Modeſty, take the right hand of* Antiquity, *and ſtand as well the* Wonder, *as* Envy *of* Poſterity: *and ſo gratefully acknowledged by all whoſe Studies and Ingenuity have qualified them with Judgement enough to profound the ſenſe of his* Geometry *and* Algebra. *And its SUBJECT ſo univerſally* Gratefull; *that I dare ſay, you have not, in all your Readings, met with the Name of any Perſon, except onely* Tacitus *the Emperour, who was ſo rude and harſh of Diſpoſition, as to diſlike the* Melody *of* Numbers.

Concerning the AUTHOR, therefore,

the

the most your ſelfe can judge me fit to ſay, is only this; that the moſt becoming Tribute I can pay unto his Noble Memory, *is a ſilent* Veneration: *it being almoſt of Neceſſity, that a* Panegyrick *on Him from my unequall Pen, be interpreted a kind of implicite* Diminution; *ſince it muſt ſuppoſe the* Height *of His* Merit *to be commenſurable by the* Digits *of ſo ſlender a* Capacity; *and few will admit Him for a Competent* Doxologiſt, *who is, by incomputable diſtances, below a due Apprehenſion of the* Excellences *of his* Subject.

And, as for the SUBJECT likewiſe, wherewith the Rationall Soule of Man *is ſo Pathetically, and by a kinde of occult* Magnetiſme, *Affected, that even the moſt* Rigid *and* Barbarous *have ever Confeſt it to be the moſt potent Charme either to* Excite, *or* Compoſe *the moſt vehement* Paſſi-

ons *thereof*; *as* Homer *ingeniously intimates in his Figment, that it was the Custome of the* Gods, *to pacifie their Civil Dissentions with the Harmony of Musick, and that the Rough spirited* Achilles, *with the soft Concordant* Echoes *of his owne* Harp, *used to Calme the tumultuous æstuation of his* Choler; *and as all* Poets *unanimously intend, in that they have made the* Magick *of* Sirens *to consist only in the* sweet Accents *and* Melotheticall Modulation *of their* Voices: *Concerning this, I say, it would sound a mere* Pleonasme *for me, here, to* Commend *it by any other* Argument, *but this unfrequent one. That the Sage and Upright* Ancients *had* Musick *in so high* Estimation, *as that, when they would fully Characterise a* Learned *and* Sapient *Person, they called him only* ἀνὴρ μουσικὸν, *a* Musician: *and, if his long Study of* Humanity

ty *and the* Liberall Sciences *had raiſed* Him *to* Eminency; *they onely went two* Notes *higher, and in the ſuperlative degree ſtyled* Him μουσικώτατον, *as if to be well skilled in the* Concordant *and* Diſcordant Proportions *of* Numbers, *were the moſt perfect* Diapaſon *of* Virtue *and* Knowledge. *Thus much, beſides the expreſſe Records of* Plutarch *and* Diogenes Laertius, *may be naturally inferred from hence; that even the beſt of our Moderne* Grammarians, *and* Philologers *derive the word* Muſick, *as alſo the* Muſes, *from the Greeke Verbe,* μάω, *that ſignifies to* Explore with deſire: *and this, upon no ſlender Reaſon; inſomuch as the Key that opens the difficult* Locks *of all* Arts *and* Sciences, *muſt be an ardent* Deſire of Diſquiſition. *The ſame alſo may bee eaſily Collected from this Conſideration; that to a* Complete Muſitian

(*pleaſe*

(*pleaſe you, to underſtand Him to be ſuch, as hath not only Nibbled at, but ſwallowed the whole* Theory *of Muſick*; i. e. *having profoundly ſpeculated the* Pythagorean *Scheme of the various Sounds ariſing from various Hammers, beaten on an Anvill, reſpective to their different* Weights, *doth clearly and diſtinctly underſtand as well the* Arithmetical, *as* Geomtrical *Proportions of* Conſonances, *and* Diſſonances: *for, it is not the mere* Practical *Organiſt, that can deſerve that Noble Attribute*) *is required a more then ſuperficial inſight into all kinds of Humane Learning. For, He muſt be a* Phyſiologiſt; *that He may demonſtrate the Creation, Nature, Proprieties, and Effects of a Natural Sound. A* Philologer, *to inquire into the firſt Invention, Inſtitution, and ſucceding Propagation of an Artificial Sound, or Muſick. An* Arithme-

Arithmetician, *to be able to explaine the Causes of Motions Harmonical, by Numbers, and declare the Mysteries of the new Algebraical Musick.* *A* Geometrician; *to evince, in great variety, the Original of Intervalls Consono-dissonant, by the Geometrical, Algebraical, Mechanical Division of a Monochord.* *A* Poet; *to conform his Thoughts, and Words to the Lawes of præcise Numbers, and distinguish the Euphonie of Vowells and Syllables.* *A* Mechanique; *to know the exquisite Structure or Fabrick of all Musical Instruments, Winde, Stringed, or Tympanous* aliàs *Pulsatile.* *A* Metallist; *to explore the different Contemperations of Barytonous and Oxytonous, or Grave and Acute toned Metalls, in order to the Casting of tuneable Bells, for Chimes,* &c. *An* Anatomist; *to satisfie concerning the Manner, and Or-*

gans of the Senſe of Hearing. *A* Melothe-tick ; *to lay down a demonſtrative method for the Compoſing, or Setting of all Tunes, and Ayres. And, laſtly,* He *muſt be ſo far a* Magician, *as to excite Wonder, with reducing into Practice the Thaumaturgical, or admirable Secrets of Muſick : I meane, the Sympathies and Antipathies betwixt Conſounds and Diſſounds ; the Medico-magical Virtues of* Harmonious Notes (*inſtanced in the Cure of* Sauls *Melancholy fitts , and of the prodigious Venome of the Tarantula ,* &c.) *the Creation of Echoes , whether Monophone , or Polyphone,* i. e. *ſingle or Multiplied, together with the Figures of Buildings , and arched Rocks, neer Rivers , Dales , or Woods, requiſite to the multiplyed Reverberations of Sounds ; the Artifice of Oto-couſtick Tubes , or Auriculary Meanders,*

for

for the ſtrengthning, continuation, and remote tranſvection of weake ſounds, and the mitigation of ſtrong; the Model of Autophonous, or ſpeaking Statues; and, finally, the Cryptological Muſick, whereby the ſecret Conceptions of the mind may be, by the Language of inarticulate Sounds, communicated to a Friend, at good diſtance.

Theſe Conſiderations præmiſed; All that can remain to me, as the proper Argument *of this* Præface, *is to adviſe you, in a word,* (1) *That the many and groſſe* Defects *obſerved in the* Latine *Impreſſion, eſpecialy in the* Figures, *and* Diagramms, *wherein the Evidence of each reſpective Demonſtration ought to have conſiſted; was a principal* Occaſion *to this my* Engliſh *one: which I may juſtly affirme to be ſo* Accurate, *ſome few* Litteral *Overſights of the* Preſs *only excepted, that the Excellent*

Des-Cartes, *had He lived to see it, would have acknowledged the* Translator *for a greater Friend to his Honour, then that rawe* Disciple *of his, who having unfaithfully transcribed the* Original, *and divulged his owne faulty* Copy; *hath often given occasion not only to the Enemies, but also some of the Defendants of his Masters Learned Industry, to suppose, that in this particular Treatise, He write some things more then Himself clearly understood. And* (2) *that the* Authour *of the concise, but weighty* ANIMADVERSIONS *subsequent, long labouring his Thoughts in the strict Examination of the Apodictical Verity of* Des-Cartes, *Fundamentals, in this* Compendium; *most happily lighted on the* Discovery *of a* New Hypothesis, *demonstratively sufficient to the full and easie [illegible] all the* Phænomena *in Musick:*

a Summary whereof, I doe here, as well to prepare, as endear your Attention, præsent you.

All Consonances, *and other Musical* Intervalls *doe arise*

According to Des-Cartes *Principles, from an* Arithmetical *Division of the Chord*, i.e. *by Dichotomising the space of an* Eighth, &c. *as an Eighth from a Bipartition of the whole Line.*

According to others, and the most Judicious Writers on this Subject (*such are* Mersennus, Lib. de Instrum. Harmonic. i. propos. 15. & Kircherus, in Artis magn. Consoni & Dissoni Lib. 4.) *from the Division of an Eighth* Geometrically, i. e. *into twelve equal* Semitones, *by eleven meane Proportionals.*

But, according to the New *Supposition excogitated by the profound Authour of*

theſe Animadverſions; from the Diviſi-on of the whole Chord into Extreame *and* Mean Ration, *and of the Mean Ration, according to this Analogie,* Viz.

As the Number of Parts *in the* Firſt *Terme,*
to the Number of Parts in the Third:
So the Number of Rations *between the* Firſt *and* Second,
to the Number of Rations between the Second *and* Third.

Which Novell Invention *alone, is more then enough, on the one ſide, to give the Capable part of Scholers a gratefull* Reliſh *of the* Inventors *extraordinary Abilities in the Nobleſt Member, or Heart of Learning the* Mathematicks: *ſo alſo, on the other, to promiſe an advantageous Compenſation of ſo ſmall an expence of Oyle, as is required*

to

to the comprehenſive perulal (*not to take notice of the contemptible* Price) *of theſe few* Sheets. *In the Confidence whereof, it is fit I ſurrender you to the pleaſant Lecture and Enjoyment of the* Book *it ſelf.*

A Compendium of Musick.

CHAPTER I.

THe *OBJECT* of this Art is a Sound.

The *END*; to delight, and move various Affections in us. For Songs may bee made dolefull and delightfull at once: nor is it strange that two divers effects should result from this one cause, since thus Elegiographers and Tragœdians please their Auditors so much the more, by how much the more griefe they excite in them.

The *MEANS* conducing to this End, or the Affections of a Sound are chiefly two; *viz.* the Differences therof in the reason of Duration or Time, and in the reason of its intension or modification into Acute or Grave; for concerning the quality of a Sound, from what body and how it may procede more gratefull, is the Argument of Physiologists.

This only thing seems to render the voice of Man the most gratefull of all other sounds; that it holds the greatest conformity to our spirits. Thus also is the voice of a Friend more gratefull then of an Enemy, from a sympathy and dispathy of Affections: by the same reason, perhaps, that it is conceived that a Drum headed with a Sheeps skin yeelds no sound, though strucken,

if another Drum headed with a Wolfs skin bee beaten upon in the ſame Room.

CHAP. II.

Præconſiderables.

1. EAch Senſe is capable of ſome Delectation.

2. To this Delectation is required a certain proportion of the object to the ſenſe. Hence comes it, (for inſtance) that the noiſe of Thunder, and the report of Guns are not convenient to Muſick : becauſe they offend the Ear, as the too great ſplendor of the Sun doth deſtroy the ſight.

3. The Object muſt bee ſuch, as that it fall not upon the Senſe with too great Difficulty and Confuſion. Hence comes it, (for inſtance) that any Figure exceedingly implicate, though exactly regular, ſuch is the Mother in the Aſtrolabe, is not ſo pleaſant to the Aſpect, as another conſiſting of lines more equall; ſuch as is in the ſame Net : the reaſon wherof is, becauſe the ſenſe doth more fully ſatisfie it ſelf in the one, then in the other, wherin are many things which it doth not perceive ſufficiently diſtinct.

4. That Object is more eaſily perceived by the ſenſe, in which is found the leaſt Difference [1] of Parts. [1]

5. The parts of an Object are ſaid to bee leſſe different each from other, when they mutually hold the greater proportion [2] each to other. [2]

6. That proportion ought to be *Arithmeticall*, not Geometricall. The reaſon wherof is, becauſe, in that, there

there are not ſo many things advertible, ſince the Differences are every where equall: and therfore the ſenſe ſuffers not ſo much labour and defati-
gation, that it may diſtinctly perceive
all things occurring therin [3]: For 2 [3]
example, the proportion of theſe lines 3
is more eaſily diſtinguiſhed by the eies, 4
then of theſe

a
[4] 4/3 [4]
4 a b c

becauſe in the firſt, the ſenſe is required only to advert the Unity for the difference of each line; but in the ſecond, the parts AB, and BC, which are incommenſurable. And therfore, I conceive, they can by no means be perfectly perceived by the ſenſe, together and at once, but only in order to a proportion *Arithmeticall*; ſo that it may advert in the part AB two parts, [5] wherof three [6] [5]
are exiſtent in BC; wherin it is manifeſt, that the ſenſe [6]
is perpetually deceived.

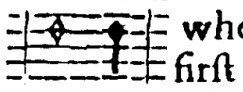
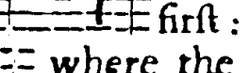
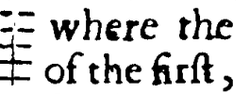
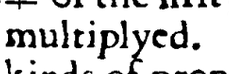
7. Among Objects of the ſenſe, that is not moſt gratefull to the Mind, which is moſt eaſily perceived by the ſenſe; nor that, on the contrary, which is with the moſt difficulty apprehended: but that which is perceived not ſo eaſily, as that that naturall deſire, wherby the ſenſes are carried towards their proper Objects, is not therby totally fulfilled; nor yet ſo hardly, as that the ſenſe is therby tired.

8. Finally, it is to be obſerved, that *Variety*, is moſt gratefull in all things. Theſe Propoſitions conceded, let us conſider the firſt *Affection* of a Sound.

CHAP. III.

Of Number, or Time to be obſerved in Sounds.

Time, in Sounds, ought to conſiſt of equall Parts; becauſe ſuch are the moſt eaſily of all others perceived by the ſence, (according to the fourth Præconſiderable:) or of Parts which are in a double or triple proportion, nor is there any further progreſſion allowable; becauſe ſuch are of all others the moſt eaſily diſtinguiſhed by the ear, (according to the fifth and ſixth Præconſiderables.) For, if the meaſures were more unequall, the Hearing could not apprehend their differences without labour and trouble, as experience witneſſeth: For, if againſt one note we ſhould place (for inſtance) five equall ones; it could not be ſung without extream difficulty.

You object, that four Notes may be placed againſt one, or eight; and therefore a farther progreſſion may be made to theſe Numbers. We anſwer, that theſe Numbers are not the firſt among themſelves, and therefore doe not generate new proportions; but only multiply a double: which is conſtant from hence, that they cannot be ſet unleſſe combinated, nor can we ſet ſuch
[7] Notes [7] alone,  where the ſecond is the fourth part of the  firſt:
But thus,  where the laſt ſeconds are the half part  of the firſt, and ſo there is only a double proportion multiplyed.

From theſe two kinds of proportions in Time, there ariſe

ariſe two kinds of Meaſures in Muſick: namely by a Diviſion into Three in time, or into Two. But, this Diviſion is noted by a percuſſion, or ſtroke, as they call it; which is ordained to aſſiſt our Imagination, that ſo we may the more eaſily perceive all the members of the Tune, and be delighted with the proportion, which ought to be in them. Now, this proportion is moſt frequently kept in the members of the Tune, in order to the helping of our Imagination', ſo that while we yet heare the laſt of the time,we may remember what was in the firſt,and what was in the reſt of the Tune. Which is effected, if the whole Tune be compoſed of 8, or 16, or 32,or 64, *&c.* members: ſo that all Diviſions may proceed from a double proportion. For then, when we have heard the Two firſt members,we apprehend them as one, while yet wee conjoyne the Third member with the Firſt, ſo that the proportion becomes triple: afterward, when we have heard the Fourth, we conjoyn it with the Third, and ſo apprehend it as one and the ſame. Then we again conjoyn the Two Firſt with the Two Laſt, and ſo apprehend thoſe Four together as One. And thus doth our Imagination proceed even to the end: where at length it conceives the whole Tune, as one intire thing compoſed of many equall members.

Few have underſtood, how this Meaſure can be exhibited to the ears without a percuſſion, or ſtroke, in Muſick, very diminute and of many voyces. This we ſay is effected only by a certain intenſion of the *Spirit* or breath, in *Vocall* Muſick; or of the *Touch*, in *Inſtrumental*: ſo as from the beginning of each ſtroke, the ſound is emitted more diſtinctly. Which all Singers naturally obſerve, and thoſe who play on Inſtruments; princi-

pally in Tunes, at whoſe numbers we are wont to dance and leap: for, this Rule is there kept, that we may diſtinguiſh every ſtroke of the Muſick, with a ſingle motion of our bodies; to the doing of which we are alſo naturally impelled by Muſick. For certain it is, that a ſound doth concuſſe, or ſhake all circumjacent bodies, as is exemplified in *Thunder*, and the ringing of *Bells*; the reaſon whereof is to be referred to the diſquiſition of *Phyſiology*. But, inſomuch as the *Hoti* is confeſt by all men, and that the ſound is emitted more ſtrongly, and diſtinctly in the beginning of each Meaſure, as we have formerly hinted: we may well affirm, that that ſound doth more ſmartly and violently concuſſe or agitate our Spirits, by which we are excited to motion; as alſo by conſequence, that *Beaſts* may dance to number, or keep time with their Feet, if they be taught and accuſtomed thereto; becauſe to this, nothing more is required, then only a mere naturall *Impetus*, or pleaſant violence.

Now, concerning thoſe various Affections, or Paſſions, which Muſick, by its various Meaſures can excite in us; we ſay, in the Generall, that a ſlow meaſure doth excite in us gentle, and ſluggiſh motions, ſuch as a kind of Languor, Sadneſſe, Fear, Pride, and other heavy, and dull Paſſions: and a more nimble and ſwift meaſure doth, proportionately, excite more nimble and ſprightly Paſſions, ſuch as Joy, Anger, Courage, *&c.* The ſame may be alſo ſayd of the double kind of percuſſion, *viz.* that a *Quadrate*, or ſuch as is perpetually reſolved into equals, is ſlower and duller, then a *Tertiate*, or ſuch as doth conſiſt of Three equal parts. The reaſon whereof is, becauſe this doth more poſſeſſe and imploy the ſence, inaſmuch as therein are more (namely 3) members to be

be adverted; while in the other are only 2. but a more exact & ample disquisition of this rare secret, doth depend upon the exquisite cognition of the *Motions* of the *Minde*; of which this place is uncapable.

However, we shall not omit, that so great is the force of *Time* in Musick, as that it alone can of it selfe adfer a certain *Delectation*; as is experimented in that Military Instrument, the *Drum*, wherein nothing else is required then meerly measure of Time; which therefore (I conceive) cannot there be composed of only 2, or 3 Parts, but also of 5, or perhaps 7 others. For since in such an Instrument the sence hath nothing else to take notice of, but bare Time: therefore in Time may be the greater *Diversity*, that so it may the more exercise and imploy the sence.

CHAP. IV.

Of the Diversity of Sounds, concerning Acute and Grave.

THis may be considered chiefly in three manners, or wayes; either in sounds which are emitted at once and together from divers bodies; or in those which are emitted successively from the same voyce; or lastly, in those which are emitted successively from divers voyces, or sonorous bodies. From the first manner, arise *Consonancies*: from the second, *Degrees*: from the third, *Dissonancies*, which come nearer to Consonancies. Where it is manifest that in Consonancies the Diversity of Sounds ought to be lesse, than in Degrees; because that would more tire, and disgust the Hearing in

in ſounds, which are together emitted, then in thoſe that are emitted ſucceſſively. The ſame alſo, in proportion, may be affirmed concerning the Difference of Degrees from ſuch Diſſonancies, as are tolerated in relation.

CHAP. V.

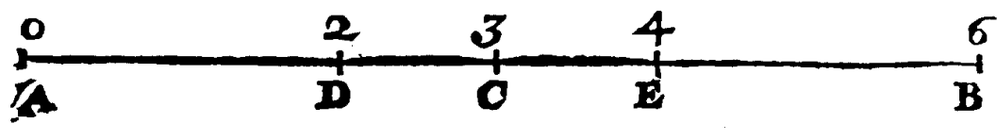
Of Conſonancies.

FIrſt, we are to obſerve, that an Uniſon is no Conſonance; becauſe therein is no Difference of Sounds, as to Acute and Grave: but that it bears the ſame relation to Conſonances, that Unity doth to Numbers.

Secondly, that of two Terms, required in Conſonances, that which is the more Grave, is far the more Potent, and doth in a manner contain the other Term in it ſelfe: as is manifeſt in the Nerves of a Lute, of which when any one is percuſſed, thoſe ſtrings, which are an Eighth, or Fifth more acute [8], tremble and reſound of their own accord; but thoſe which are more Grave do not, at leaſt do not appear to the ſence ſo to do; the Reaſon whereof is thus demonſtrated. *One ſound bears the ſame reſpect to another ſound, that one ſtring bears to another ſtring:* but in every ſtring that is greater, all the other ſtrings, that are leſſe, are comprehended; though every ſtring that is longer, doth not comprehend all the others, that are ſhorter: and therfore alſo in every Graver Sound, all others more Acute are comprehended; but not, on the contrary, in every Acuter Sound are the more Grave comprehended: whence it is evident, that the

[8]

the more Acute Termis to be found by the Diviſion of the more Grave. Which Diviſion that it ought to be Arithmeticall, *i.e.* into equall parts, is conſequent from what was before obſerved in the ſixth Præconſiderable.



Let, therfore, *A B* bee the more Grave Term, in which if I would find the Acuter Term of all the firſt Conſonances, I muſt divide it by the firſt of all Numbers, *viz.* by 2, as is done in *C*; and then *A C*, *A B*, are diſtant each from other, the firſt of all the Conſonances, which is called an Eighth and Diapaſon. Further, would I have other Conſonances, which immediately follow the firſt; I muſt divide *A B* into three equall parts; and then I ſhall have not only one Acute Term, but two, *viz.* *A D*, and *A E*, from which there will ariſe two Conſonances of the ſame kind, *viz.* a Twelfth, and a Fifth. Again, I can ſubdivide the line *A B* into 4, or 5, or 6 parts, but no further; becauſe ſuch is the imbecillity of the Ears, as that they cannot diſtinguiſh, without ſo much labour as muſt drown the pleaſure, any more Differences of Sounds [9]. [9]

Heer we are required to note, that from the firſt Diviſion doth ariſe only one Conſonance: from the ſecond, two: from the third, three: as this Table demonſtrateth [10]. [10]

First Figure.

$\frac{1}{2}$	Eigh th									
$\frac{1}{3}$	Twelfth	$\frac{2}{3}$	Fifth							
$\frac{1}{4}$	Fifteenth	$\frac{2}{4}$	Eighth	$\frac{3}{4}$	Fourth					
$\frac{1}{5}$	Sevēteenth Major	$\frac{2}{5}$	Tenth Major	$\frac{3}{5}$	Sixth Major	$\frac{4}{5}$	Ditone			
$\frac{1}{6}$	Nineteenth	$\frac{2}{6}$	Twelfth	$\frac{3}{6}$	Eighth	$\frac{4}{6}$	Fifth	$\frac{5}{6}$	Third Minor	

Heere wee have not set downe all Consonances that are; in regard, that, to our more facile Invention of the rest, requisite it is that we first treat

CHAP.

CHAP. VI.

Of an Eighth.

THat this is the firſt of all Conſonances, and that which is the moſt eaſily perceived by the Hearing after an *Uniſon*; is manifeſt from the Premiſes, and alſo comprobated by experiment in Pipes: which, when blown with a breath ſtronger than ordinary, inſtantly yield a ſound more Acute one Eighth. Nor is there any reaſon, why that ſound ſhould immediately ariſe to an Eighth, rather than to a Fifth, or any other Note; unleſſe becauſe an Eighth is the firſt of all Conſonances, and that which is the leaſt different from an Uniſon. From whence, we conceive, it doth alſo follow, that no ſound can be heard, but it ſeems in ſome ſort to reſound in the ear more Acute an Eighth: and that this is alſo the cauſe, why in a Lute to the greater ſtrings, which give Graver ſounds, other ſmaller ſtrings more Acute one Eighth are conſociated, which are alwayes percuſſed at the ſame inſtant, and ſo effect that the Graver ſounds are heard more diſtinctly. Whence it is manifeſt, that no ſound which ſhall be conſonant to one Term of an Eighth, can be diſſonant to any other Term of the ſame Eighth.

A *ſecond* thing to be obſerved concerning an Eighth, is this; that it is the greateſt of all Conſonancies, that is, that all other Conſonancies are contained therein; or compoſed [11] therof, and of others which are contained therein. Which may be demonſtrated from hence, that [11]

[12] all Conſonancies conſiſt of equall parts [12]; whence it comes, that if their Terms be more diſtant each from other than one Eighth, we may, without any further Diviſion of a more Grave Term, adde one Eighth to a more Acute, of which, together with the reſidue, it will [13] appear that that is compoſed [13]. An Example may be *A B*, divided into three equall parts, of which *A C*, *A B*, are diſtant by one Twelfth: we ſay, that Twelfth is compoſed of an Eighth, and the reſidue thereof, *viz.* [14] a Fifth [14]; for compoſed it is of *A C*, *A D*, which is

an Eighth; and *A D*, *A B*, which is a Fifth: and ſo it falls out in the reſt. Whence it comes, that one Eighth doth not ſo multiply the numbers of proportion if it compoſe others, as all others do; and is therefore the only Conſonance which is capable of *Gemination*, or Doubling. For, if it be Geminated, it makes only 4 [15] [15], or 8, if regeminated: but if a Fifth be Geminated, which is the Firſt after an Eighth, it makes 9 [16] [16]: for from 4, to 6, is a Fifth; in like maner from 6, to 9; which number is far greater then 4, and exceeds the ſeries of the firſt ſix Numbers, in which we have [17] formerly included all Conſonances [17].

From this it naturally follows; that of all Conſonancies, of what kind ſoever, there are but three Species: one is Simple: another Compound of a Simple and an Eighth: a third compoſed of a ſimple and 2. Eighths. Nor can any other Species be added, which is compoſed of 3 Eighths, and another ſimple Conſonance; becauſe theſe are the extream limits, nor is there

there any progreſſion beyond three Eighths; ſince then the numbers of Proportions would be multiplyed exceſſively. From whence is deduced a generall Catalogue of all Conſonances whatever, which is here preſented in the following Table.

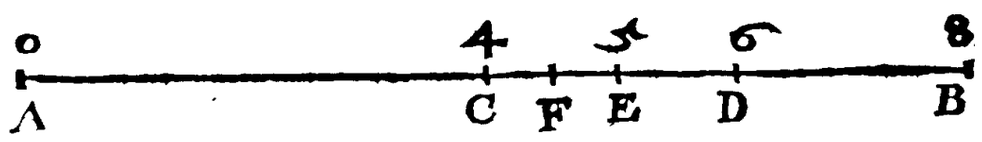
Second Figure.

	Simple Conſonances.	*Firſt Compound Conſonances.*	*Second Compound Conſonances.*
Eighths	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Fifths	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
Ditones	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{1}{5}$
Fourths	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$
Sixths majors	$\frac{3}{5}$	$\frac{3}{10}$	$\frac{3}{20}$
Thirds minors	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{5}{24}$
Sixths minors	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{5}{32}$

Here have we added the *Sixth Minor*, which we had not obſerved in the precedent Chapter; in regard it may be educed from what hath been ſayd of an Eighth, from which if a Ditone be cut off, the remainder will
[18] be a Sixth Minor [18]. But of this more clearly anon.

Wheras we even now affirmed, that all Conſonances
[19] were comprehended in an Eighth [19]; we are concerned to inquire how that comes to paſſe, and how they proceed from the Diviſion thereof, that ſo their nature may be the more plainly and diſtinctly underſtood.

Firſt, it is moſt certain, that that Diviſion of an Eighth, from which all Conſonances ariſe, ought to be Arithmeticall, or into equall parts: now what that is, which ought to be divided, is evident in the ſtring *A B*, which is diſtant from *A C*, the part *C B*; but the



ſound *A B*, differs from the ſound *A C*, an Eighth: therefore will the ſpace of an Eighth be the part *C B*. That therefore is it, which ought to be divided into two equalls, that the whole Eighth may be divided, which
[20] is effected in *D* [20]. From which Diviſion, that we may underſtand what Conſonance is properly, and *per ſe* generated; we are to conſider that *A B*, which is the more grave Term, is divided in *D*, not in order to it ſelf, for then it would have been divided in *C*, as was done before: nor, as the Caſe ſtands now, is an Uniſon divided,

ded, but an Octave, which conſiſts of two Terms, and therefore when the more Grave Term is divided, that Diviſion is made in order to another more Acute. Whence it comes that the Conſonance properly ariſing from the Diviſion, is between the Terms *A C*, *A D*, which is a Fifth; not betwixt *A D*, *A B*, which is a Fourth: becauſe the part *D B*, is only the reſidue, and generates a Conſonance by accident; from hence, that ſound which makes a Conſonance with one Term of an Eighth, ought alſo to make a Conſonance with the other.

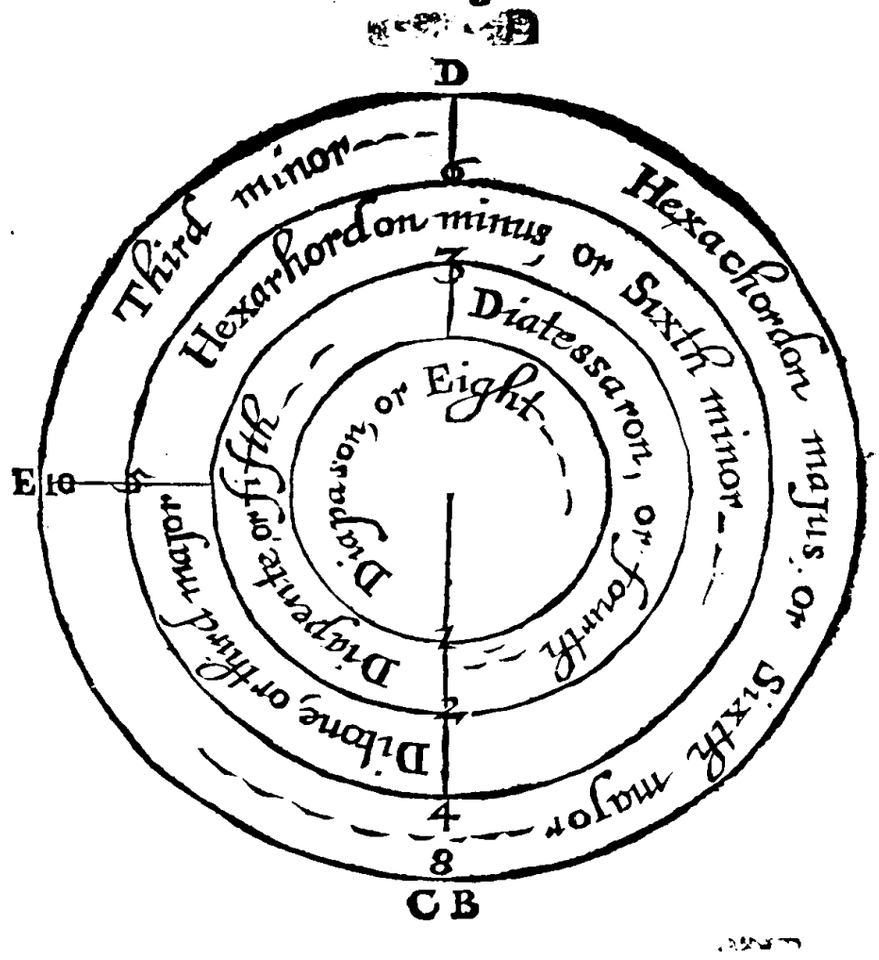
Again, the ſpace *C B* being divided in *D*, I might by the ſame reaſon divide *C D* in *E* [21]; from whence a Ditone would be directly generated, and by accident all the other Conſonances: nor is it requiſite that *C E* be further divided; yet if that were done, *viz.* in *F* [22], then would from thence ariſe a greater Tone, and by accident a leſſer Tone, and the Semitones [23], of which hereafter: for, in a voyce, they are ſucceſſively admitted, but not in Conſonances. [21] [22] [23]

Nor let any think it imaginary, what we ſay, that only a Fifth and a Ditone are generated from the Diviſion of an Eighth properly, and all other Conſonances by Accident; for Experience teacheth the ſame in the ſtrings of a Lute or other Inſtrument, whereof if one be ſtroke, the force of that ſound will ſtrike all the other ſtrings which ſhall be more Acute in any kind of Fifth or Ditone: but in the others which are diſtant a Fourth, or other Conſonance, the ſame ſhal not happen. Which force of Conſonances muſt undoubtedly ariſe from hence

hence, either from their Perfection, or Imperfection, inſomuch as theſe are firſt Conſonances of themſelves, but all others are only by Accident, becauſe they neceſſarily flow from others.

But let us enquire, whether that be true, which we formerly ſayd, *Viz.* That all Simple Conſonances are comprehended in an Eighth: this we ſhall eaſily juſtifie, if we ſhall turn *CB*, the halfe of *AB*, which contains an Eighth, into a Circle; ſo that the poynt *B* may be joyned to the poynt *C*. Then let the Circle be divided in *D* and *E*, as it was divided in *CB*: and the reaſon why all the Conſonances ought ſo to be found out, is becauſe no ſound can be conſonant to one Term of an Eighth, but it muſt alſo be conſonant to the other Term of the ſame, as we have already proved. From whence it comes, that if in the ſubſequent Figure one part of the Circle make a Conſonance; the reſidue muſt alſo contain ſome Conſonance.

Third Figure.



From this Figure it is demonstrated how rightly an Eighth is named Diapasson, because it comprehends in it selfe all the intervalls of other Consonances. Here we have exhibited only Simple Consonances; where if we would find out also Compound ones, all we are to do is only to adde, to the intervalls above described, one or two whole Circles; and then it will appear that an

Eighth doth compoſe all Conſonances.

From what hath præceded, we collect that all Conſonances may be referred to Three Kinds; for (1) either they ariſe from the firſt Diviſion of an Uniſon, ſuch are thoſe which are called Eighths, which make the Firſt Genus: or (2) they ariſe from the Diviſion of an Eighth into two equall parts, ſuch are Fifths and Fourths, which we may therefore call Conſonances of the Second Diviſion: or (3) they ariſe from the Diviſion of a Fifth, which are Cönſonances of the Third and laſt kind. We again divide them into ſuch Conſonances as ariſe from thoſe Diviſions *per ſe*; and thoſe which ariſe *per Accidens*; and that there are only three
[24] Conſonances *per ſe* [24], we have formerly ſayd, which may be confirmed from the Firſt Figure, in which we extracted the Conſonances from the Numbers themſelves: For therein we are to take notice, that there are only three ſonorous Numbers, 2, 3, and 5
[25] [25], for the number 4, and number 6, are compounded of them, and are therefore ſonorous numbers only by Accident, as doth there appear; where, in a right order and a ſtreight line, they do not generate new Conſonances, but only ſuch are compoſed from the former: for example, 4 generates a Fifteenth, and 6 a Nineteenth; but *per Accidens* and in a tranſvers line, 4 generates a Fourth, and 6 a Third leſſer; where we are to obſerve by the By, that in the Number 4, a Fourth is immediately generated from an Eighth, and is in a manner a certain Monſter, or difficient and imperfect Product of an Eighth [26].

CHAP.

CHAP. VII.

Of a Fifth.

THis, of all Conſonances, is the moſt gratefull, and acceptable to the Ear; and, for that reaſon, it is the prime and ruling Conſonance in all Tunes; as alſo from it do the *Modes* [27] proceed, as follows from the 7 *Præconſiderable*: for ſince, as it is manifeſt from what hath preceded, whether we extract the perfection of Conſonances from *Diviſion*, or from *Numbers* [28]; there can properly be found only three Conſonances, among which the fifth hath the middle place: this (certainly) is it which reſounds in the ears not ſo ſharply as a *Ditone*, nor ſo languid as a *Diapaſſon*, but the moſt pleaſant of all others. Further, from the *Second Figure* it appears, that there are three kinds of a Fifth [29], where the Twelfth poſſeſſes the mean place, which we may therefore affirm to be the moſt perfect Fifth: from whence it follows, that we might uſe no other Conſonance in Muſick, if it were not, as we inferred in the laſt Præconſiderable, that Variety was neceſſary to *Delectation*.

[27] [28] [29]

If it be objected, that, in ſome caſes, an Eighth may be ſet alone in Muſick, without any Variety; as, for Example, when two voyces ſing the ſame Tune, one more acute than the other in an Eighth: but the ſame doth not evene in a Fifth; and therefore it follows, that an Eighth ought to be accounted the moſt gratefull of all Conſonances, rather than a Fifth.

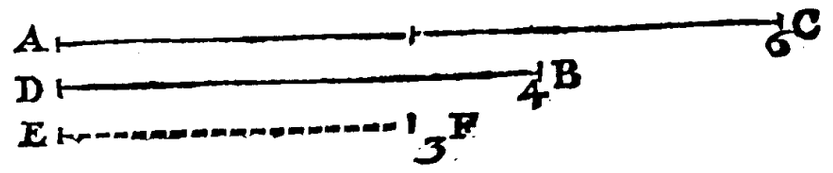
We anſwer, that, from this Inſtance, our Aſſertion is rather confirmed, than infirmed; for the reaſon, why an Eighth may be ſo ſet, is, becauſe it comprehends an Uniſon in it ſelfe, and ſo thoſe two voyces reſound in the eare as one; which happens not in a Fifth, whoſe Terms are more different among themſelves, and therefore poſſeſſe, and exerciſe the Hearing more fully; from whence a certain wearineſs and loathing would ariſe forthwith, if it were ſet alone, and without Variety in Tunes. This may be exemplified thus; we ſhould be ſooner weary if we were conſtantly fed with Sugar, and Sweat-meats, than if with bread alone; which every man will allow not, in any proportion, comparable for ſweetneſs and blandiſhment of the palate, to Sugar.

CHAP. VIII.

Of a Fourth.

THis, of all Conſonances, is the moſt unhappy; nor is it ever uſed in Tunes, unleſſe by Accident, and with the aſſiſtance of others: not that it is more imperfect than a Third Minor, or a Sixth, but that it approacheth the nature of a Fifth ſo nearly, that the grace of this is drowned in the ſweetneſſe of that. To the underſtanding of which, we are to note, that a Fifth is never heard in Muſick, but that, in ſome ſort, an acuter Fourth is withall advertiſed; which follows from [30] what we have ſayd [30], that in an Uniſon, there is, in ſome ſort, reſounded an acuter Eighth. For Example, let

let *AC* be in distance form *D B* oFi~~f~~th dne the reſo-



nance thereof, more Acute by an Eighth, be *E F*; and certainly that will be distant from *D B*, by one Fourth: whence it comes, that it may be called the ſhadow of a Fifth, which perpetually accompanies it; and thence alſo it is evident, why a Fourth cannot be ſet in Tunes, primarily, and *per ſe*, *i. e.* betwixt a Baſſe and another part. For when we ſayd, that other Conſonances were neceſſary in Muſick, only in order to the variation of a Fifth; certainly, it is evident, that a Fourth would be uſeleſſe, in regard it cannot vary a Fifth: which appears from hence; that, if it were ſet in a more Grave part, it would alway reſound more Acute than a Fifth, where the Hearing would ſoon perceive that it is deturbed from its proper place to an inferiour one, and ſo a Fourth would bee moſt harſh and unpleaſant thereto, as if only the ſhadow were preſented inſtead of the body, or the Image objected inſtead of the Thing it ſelfe.

CHAP. IX.

Of a Ditone, a Third Minor, and Sixths.

THat a *Ditone* is, by many degrees, more perfect than a Fourth, is manifeſt from the Premiſes; to which, nevertheleſſe, we ſhall adde this; that the Perfection of any Conſonance is not to be deſumded preciſely, from the ſame, while it is Simple; but alſo from all the Compounds thereof: the reaſon whereof is, that it can never be heard alone ſo jejune and empty, but the reſonance of this compoſed is alſo heard together with it; ſince that, in an Uniſon, the reſonance of a more Acute Eighth is contained, we have formerly evicted. Now, that a Ditone, ſo conſidered, doth conſiſt of leſſer Numbers than a Fourth [31], and is therefore more perfect than a Fourth; is plain from the *Second Figure*: wherein we, therefore, placed a Ditone before a Fourth, inſomuch as we endeavoured, in that Figure, to place all Conſonances according to the order of Perfection.

[31]

But here we are obliged to explain, why the third *Genus* of a Ditone is the moſt perfect, and makes, in the ſtrings of a Lute, a Tremulation perceptible even by the ſight; rather than the Firſt, or Second Genus: which we conceive to proceed from hence; that this Third doth conſiſt in a *multiplyed Proportion*, but the Firſt in a ſuper-particular, the Second in a multiplyed and ſuper-particular, together [32]. And why, from multiplyed proportion, the moſt perfect Conſonances do ariſe; which we therefore placed in the Firſt order of the

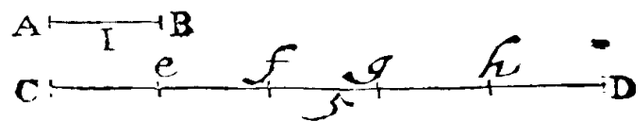
[32]

Firſt

First Figure, we thus demonſtrate.

Let the Line *A B* be diſtant from *C D*, in the Third Genus of a Ditone, howſoever men ſhall imagine the ſound to be perceived by the Hearing; certain it is that it is more eaſie to diſtinguiſh what is the pro-

For Example,



portion betweene *A B* and *C D*, than betweene *C F* and *C D*; beeauſe it will firſt bee knowne directly by the application of the ſound *A B*, to the parts of the ſound *C D*, *viz. C e*, *e f*, *f g*, *&c.* nor will there be any reſidue in the end: which falls not alike in the proportion of the ſound *C f*, to *C D*; for if *C f* be applyed to *f h*, there will be the reſidue *h D*, by the reflection of which we ought to know what is the proportion between *C f* & *C D*, which is more difficult or tedious. By the ſame way will it be conceived, if any ſay that a ſound doth ſtrike the ears with many percuſſions or verberations, and that by ſo much the more ſwiftly, by how much the more acute the ſound is; for then, that the ſound *A B* may arrive at the requiſite Uniformity with the ſound *C D*, it ought to ſtrike the ears with only five touches or verberations, while *C D* ſtrikes only once: but the ſound *C f* will not ſo ſoone returne to an Uniſonance, for that cannot be done but after the ſecond ſtroke of the ſound *C D*, as is deſcribed in the ſuperiour Demonſtration. The ſame will alſo be explained, however we conceive the ſound to be heard.

A *Third Minor* ariſeth from a Ditone, as a Fourth from a Fifth [33], and is therefore more imperfect than [33]

a Fourth, as a Ditone, is than a Fifth. Nor is it therefore to bee excluded Muſick, ſince it is not onely not uſeleſſe, but even neceſſary, in order to the variation of a Fifth. For, ſince an Eighth is alwayes heard in an Uniſon, it cannot adſer this variety; nor a Ditone alone, (for there can be no variety unleſſe betwixt Two, at leaſt:) therfore is a Third Minor aſſociated thereto, to the end that ſuch Tunes, wherein Ditones are more frequent, may be diſtinct from ſuch as have a Third Minor very often iterated in them.

A *Sixth Major* proceeds from a Ditone, and by the ſame reaſon participateth the nature thereof, as a Tenth Major, and Seventeenth [34]: to the underſtanding of which, we are to look back upon the Firſt Figure, where, in the number Foure, are found a Fifteenth, an Eighth, and a Fourth, which is the Firſt Compound Number, and which, by a Binary, (which repreſenteth an Eighth,) is reſolved even into an Unity; whence it comes that all Conſonances generated from it, are apt and convenient for Compoſition, among which ſince a Fourth is found, (which, for that reſpect, we formerly called a Monſter, or defective Eighth;) thence doth it follow, that the ſame is not unprofitable in compoſition, where the ſame reaſons do not recur, which hinder it from being ſet alone; for then is it perfected by the adjunct, and remains no longer ſubject to a Fifth,

[34]

A *Sixth Minor* proceeds from a Third Minor, in the ſame manner as a Sixth Major doth from a Ditone [35], and ſo borrows the nature and affections of a Third Minor: nor is there any reaſon to countermand it.

[35]

Here the Series of Conſonances would Exact from us a Diſcourſe concerning their various *Virtues*, as to the excitement

excitement of *Paſſions:* but a more exact Diſquiſition of this, may be collected from the Præcedents; and it exceeds the limits of a *Compendium.* For, ſo various are they, and upon ſo light circumſtances ſupported; that, a whole Volume would not ſuffice to perfect their Theory. This, therefore, ſhall we only ſay, that the chiefeſt Variety doth ariſe from theſe four laſt; whereof a Ditone and Sixth Major are more grateſull, more ſprightfull, and exhilarating than a Third and Sixth Minor; as hath been obſerved by *Practicall Muſicians*, and may be eaſily deduced from hence, that a Third Minor is generated from a Ditone only by Accident, but a Sixth Major *per ſe*, becauſe it is no other but a Ditone Compound.

CHAP. X.

Of Degrees, or Tones Muſicall.

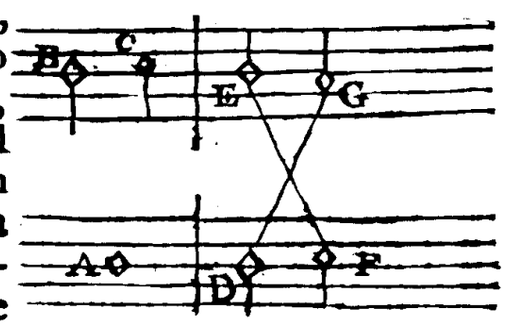
FOr two cauſes chiefly are Degrees required in Muſick; (1) That by their aſſiſtance a Tranſition may be made from one Conſonance to another, which cannot, ſo conveniently, be effected by Conſonances themſelves with Variety, the moſt gratefull thing in Muſick: (2) That all that ſpace, which the ſound runs over, may be ſo divided into certain intervals, as that the Tune may alwayes paſſe through them more commodiouſly than through Conſonances.

If we conſider them in the firſt capacity; there can be only Four kinds of Degrees, and no more: For then they ought to be deſumed from the inequality, found

between Conſonances, and all Conſonances are diſtant each from other $\frac{1}{9}$ part, or $\frac{1}{10}$, or $\frac{1}{16}$, or finally $\frac{1}{33}$ [36] beſides the intervals which make other Conſonances: therefore all Degrees conſiſt in thoſe numbers, the two firſt Tones whereof are called Major and Minor, and the two laſt are called Semitones, Major and Minor. But we are to prove that Degrees, conſidered in this capacity, are generated from the inequality of Conſonances; which is thus done. So often as there is a tranſition made from one Conſonance to another, either one Term is moved ſingle, or both together; and by neither of theſe two ways can any ſuch tranſition be made, unleſſe by thoſe intervals, which deſign the inequality betwixt Conſonances: Therefore. The firſt part of the Minor is thus demonſtrated.

[36]

[37] [37] Let from *A* to *B*, be a Fifth; and from *A* to *C*, be a Sixth Minor; and, of neceſſity, from *B* to *C* will be that difference, which is betwixt a Fifth and a Sixth Minor, *viz.* $\frac{1}{10}$, as is evident [38]: but that the Poſterior part of the Minor may be proved, wee are to obſerve; that wee are not, in ſounds, to regard only the proportion while they are emitted together, but alſo while they are emitted ſucceſſively, ſo that, as much as poſſible, the ſound of one Voyce ought to keepe Conſonance with the immediately precedent ſound of the other voyce; which can never bee effected, if the Degrees did not ariſe from the inequality of Conſonances. For Example, let *D E* be a Fifth, and let each Term be moved



[38]

moved by contrary motions, ſo that a Third Minor may be created; if *D F* be an intervall, which doth not ariſe from the inequality of a Fourth to a Fifth, *F* cannot, by relation, be conſonant to *E*; but if yea, then it can: and ſo likewiſe in the reſt, as may ſoon be experimented. Here obſerve, that as concerning that Relation, we ſayd it ought to be conſonant ſo much as poſſible: for alwayes it cannot be, as will appeare in the ſucceeding Diſcourſe.

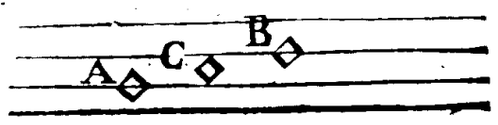
But if wee conſider them in the ſecond Capacity; namely, how theſe Degrees may, and ought to bee ordained in the whole intervall of ſounds, that by them one ſolitary voyce may be immediately elevated, or depreſſed; then, among the Tones already found oút, thoſe Degrees ſhall only be accounted Legitimate, into which the Conſonances are immediately divided. To the manifeſtation of this, wee are to advert, that every intervall of ſounds is divided into Eighths, whereof one can by no means differ from another, and therefore that it is ſufficient, if the ſpace of one Eighth be ſo divided as that all the Degrees may be obtained: as alſo, that that Eighth is already divided into a Ditone, a Third *minor*, and a Fourth [39], all which evidently follow from what wee have ſayd concerning the laſt Figure of the Superior Tractate. [39]

Hence alſo is it manifeſt, that Degrees cannot divide a whole Eighth, unleſſe they divide a Ditone, a Third *minor*, and a Fourth; which is thus done. A Ditone is divided into a Tone *major*, and a Tone *minor* [40]; a Third *minor* is divided into a Tone *major*, and a Semitone *majus* [41]; a Fourth, into a Third *minor*, and alſo a Tone *minor* [42], which Third is again divided into a [40] [41] [42]

[43] Tone *major*, and a Semitone *majus* [43]; and ſo the whole Eighth doth conſiſt of three Tones *major*, two Tones *minor*, and two Semitones *majora*; as is manifeſt to him who ſeriouſly and exactly perpends their Scheme. And here we have only three Kinds of Degrees; for a Semitone *minus* is excluded, becauſe it doth not immediately divide Conſonances, but only a Tone *minor*. As for Example, if it be ſayd that a Ditone doth conſiſt of
[44] a Tone *major*, and both Semitones [44] (for both Semi-
[45] tones compoſe a Tone *minor* [45]): but wherefore, will you ſay, is not that Degree alſo admitted, which reſulteth from the Diviſion of another, and divides Conſonances onely *Mediately*, not immediately? our Anſwer is, that the Voyce cannot run through ſo many various Diviſions, and at the ſame inſtant be conſonant with another different voyce, unleſſe with extream Difficulty, as is open to Experiment: beſides, a Semitone *minus*
[46] would then be joyned to a Tone *major* [46], with which it would create a moſt unpleaſant Diſſonance; for con-
[47] ſiſt it would between theſe numbers 64 and 75 [47], and therefore the voyce could not bee moved through ſuch an intervall. But, in order to the clearer ſolution of this Objection, we are to note;

That to the Creation of an Acute ſound, is required a more forcible emiſſion of the breath, or ſpirit in vocall Muſick; or a ſtronger percuſſion of the ſtrings in inſtrumentall; than is required to the Creation of a Grave: which is experimented in the ſtrings of a Lute, which yield a ſound by ſo much the more Acute, by how much the more they are diſtended; as alſo from hence, that by a greater force, the Aer is divided into leſſer parts, from which the more Acute ſound muſt of neceſſity

neceſſity be generated: and from hence it is a direct Conſequence; that by how much the more Acute a ſound is, by ſo much the more ſtrongly doth it ſtrike the eares. From this animadverſion, I conceive, a true and chiefe reaſon may be rendred, wherefore *Degrees were invented; viz.* leaſt, if the voyce ſhould run through the Termes of Conſonances alone, there would bee among them too great a diſproportion in the reaſon of intenſion, which would inevitably tire both the Auditors and Singers. For Example, would I aſcend from *A* to *B*, becauſe the ſound *B* wil ſtrike the ears far ſtronger, than the ſound *A*, leſt that Diſproportion ſhould be incommodious, the Term *C* is ſet in the midle, by which we may, as by a Degree, more eaſily, and without that inequall contention of the breath, aſcend to *B*. From which it is manifeſt, that Degrees are nothing elſs but a certaine *medium*, interpoſed betweene the Terms of Conſonances, for the moderation of their inequality; and that of themſelves they have not ſweetneſſe enough to ſatisfie the ears, but are only conſiderable and uſefull in order to Conſonances; ſo that while the Voyce runs through one Degree, it leaves the Hearing unſatisfied, untill it ſhall have arrived at a Second; which, for that reſpect, ought, together with the former Degree, to conſtitute a Conſonance: and this is ſufficient to ſolve the præcedent Objection. Moreover, this alſo is the reaſon, why, in a Voyce, ſucceſſively Degrees are admitted, rather than Ninths or Sevenths, (which ariſe from Degrees,) or others which do conſiſt of leſſe Numbers than Degrees; namely, becauſe intervals of this ſort do not



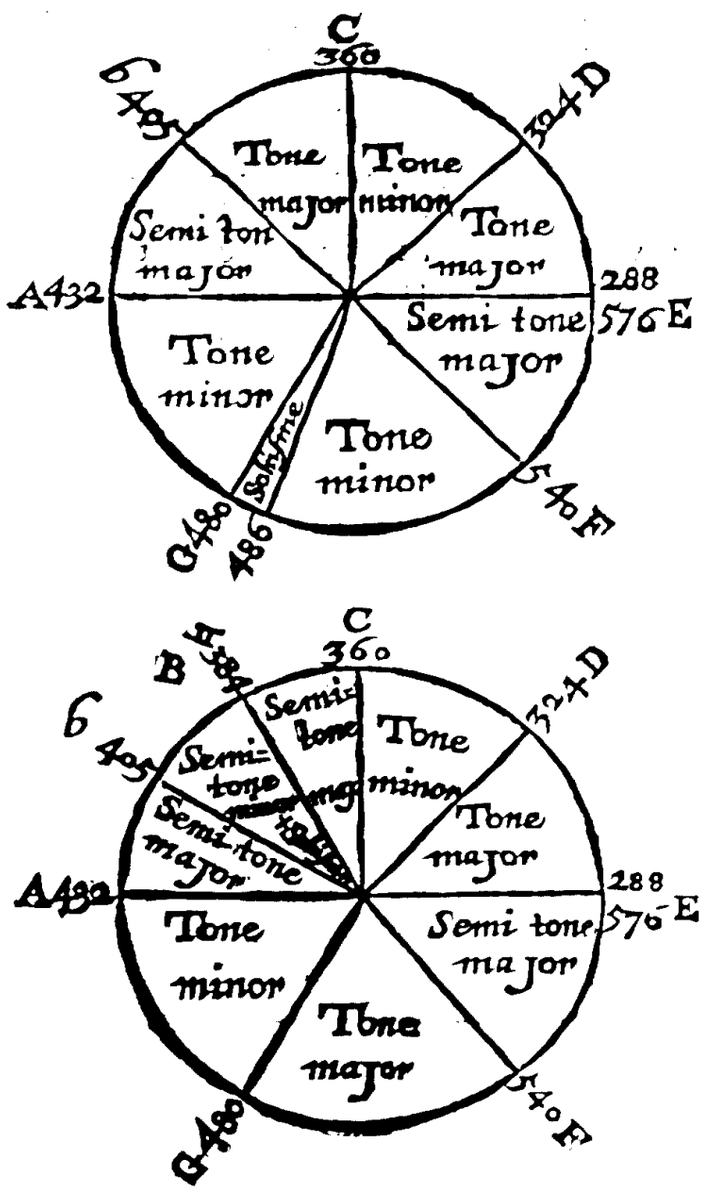
divide the least Conſonances, nor can they therfore moderate that inequality, which is betwixt their Terms. More, concerning the invention of *Degrees*, (which ariſe from the Diviſion of a Ditone into two parts, as a Ditone doth from the Diviſion of a Fifth,) might be ſuperadded; and many things from thence be deduced, which concern their ſundry *Perfections*: But it would require more room than a *Compendium* can afford, and a good Underſtanding may infer as much, from what hath præceded concerning *Conſonances*.

More requiſite it is, that, in the preſent, we ſpeak of the Method or Order, in which thoſe Degrees are to be conſtituted in the whole ſpace of an Eighth; now this Order ought to be ſuch, as that a Semitone *majus*, may have on each ſide next to it a Tone *major* [48]; as alſo a Tone *minor* [49], with which this doth compoſe a Ditone; and the Semitone a Third *minor*, according to what we have juſt now obſerved [50]: but ſince an Eighth containeth Two Semitones, and as many Tones *minor*; that this may be obtained without Fraction, it ought alſo to containe Foure Tones *major* [51]: Now becauſe it containes only three, therefore is it neceſſary, that, in ſome place, wee uſe a certaine Fraction, which may be the difference betwixt a Tone *major* and a Tone *minor*, which we nominate a *Schiſm* [52]; or alſo between a Tone *major* and a Semitone *majus*, which contains a Semitone *minus* with a *Schiſm* [53]: to the end, that by the helpe of theſe Fractions the ſame Tone *major* may, after a ſort, bee made moveable, and ſo perform the office of two Tones; which is eaſily preceptible in

[48] [49] [50] [51] [52] [53]

in the Figures here delineated, where we have turned the whole ſpace of an Eighth into a Circle, after the ſame manner, as in the end of the Sixth Chapter.

And truely in either of theſe Figures, every intervall deſigneth one Degree, except Two: *viz.* a Schiſm in the Firſt, and a Semitone *minus* with a Schiſm in the Second: which Two are in ſome ſort moveable, ſo that they may bee referred ſucceſſively to both Degrees immediately annexed unto it.


C
360
D
324
E
288
576
F
540
G
486
480
A
432
B
384
b
405
Tone major
Tone minor
Semi ton major
Tone major
Semi tone major
Tone minor
Schisme
Tone minor
Semi-tone
Semi-tone major
Tone major
Semi tone major
Tone major

Now,

Now manifest it is from these Figures, (1) That in the First Figure, there can be no ascention by Degrees from 288 [54] to 405, unlesse wee emit the middle Terme, in some sort irregular or quavering; for that if it respect 288, it may seeme to bee 480, but if it respect 405, then it may seeme to bee 486, *viz.* that with both it make a Third *minor*, and the difference is so smal betwixt 480 and 486, that the mobility of that Terme, which is continued from both, doth not strike the Hearing with a Diffonance perceptible.

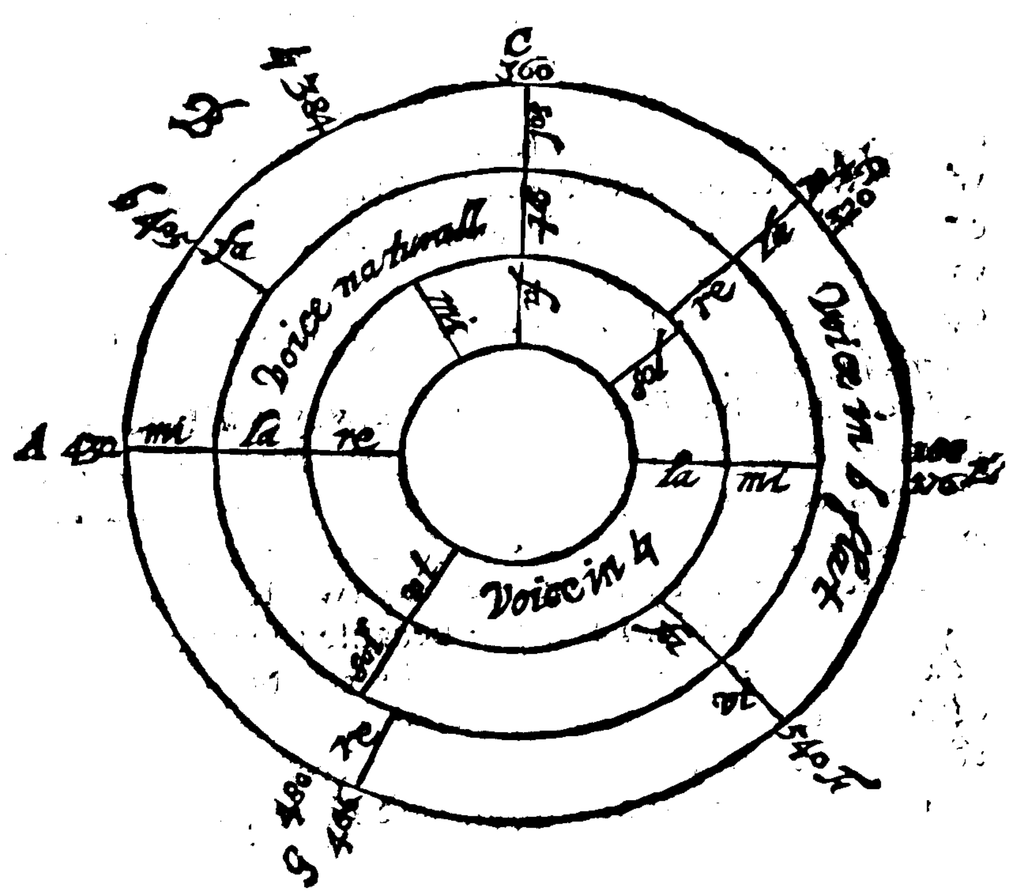
(2) In the Second Figure, after the same reason, we cannot ascend from the Terme 480 to 324 by Degrees, unlesse wee so expresse the middle Terme, as that, if it respect 480, it may seem 384; if it respect 324, it may be 405, that so, with both, it may make a Diatone. But, because betwixt 384 and 405, the difference is so great, that no voyce can be so tempered at once, as that if it hold a Consonance with one of the extreams, but it will appeare exceedingly Diffonant from the other: therefore is another way to bee sought, by which (the most of all others) this so great an incommodity may be, if not totally removed, yet at least greatly diminished. Now this can be no other way, but what is found and described in the Superiour Figure, *viz.* by the use of a Schism: by this means, if wee would goe through the Terme 405. Wee will remove the Terme *G*, by one Schism, that it may be 486, no more 480: and if wee would goe through 384, we will change the Terme *D*, and 320 shall be in the place of 324, and so shall be distant, by a Third *minor*, from 384.

From theſe conſiderations it is evident, that all the ſpaces, through which one voyce ſolitary may bee moved, are contained in the Firſt Figure: for, when the incommodity of the Second Figure is corrected, then doth it not differ from the Firſt [55]; as is eaſily deprehended. [55]

Evident it is alſo, that that Order of Tones, which practicall Muſitians call the *Hand*, doth contain all the Modes, by which Degrees may be ordained; for, that they are comprehended in the two præcedent Figures, is formerly proved: and that Hand of Practicall Muſicians doth contain all the Termes of each Precedent Figure, as is eaſily diſcerned in the following Figure, in which we have turned that Hand into a Circle, that ſo it might the better be referred to the Superiour Figures. Yet, to the underſtanding of this Figure, we are to ſignifie, that it begins from the Term *F*, where, for that cauſe, we have applyed the greateſt number, that thence it might be collected that that Term is of all the moſt *Graue* [56]. [56]

Figure

Figure the Sixth.



That it ought to be ſo, is inferred from hence; that wee can begin Diviſions from onely two places of the whole Eighth: ſo that therein either two Tones may be ſet in the firſt place, and, after one Semitone, three Tones conſequent in the laſt place; or, on the contrary, three Tones in the firſt place, and only two in the laſt. And the Term *F* repreſenteth both thoſe two places to-

 gether

gether. For, if from thence we go by *b*, only two Tones, are in the first place, but if by *c*, there will bee three: Therefore.

First, then it is manifest from this Figure, & the second precedent, that onely Five Spaces are contained in a whole Eighth, by which the voyces can naturally proceed, *i. e.* without any Fraction, or moveable Terme, which was to bee found out by Art, that it might proceed further. Whence it came, that those five intervalls should be attributed to a Naturall Voyce, and six only Voyces were found out to expresse them; *viz. ut, re, mi, fa, sol, la.*

Secondly, that from *ut* to *re*, is always a Tone *minor*; from *re* to *mi*, always a Tone *major*; from *mi* to *fa*, always a Semitone *majus*; from *fa* to *sol*, always a Tone *major*; and lastly from *sol* to *la*, a Tone *minor*.

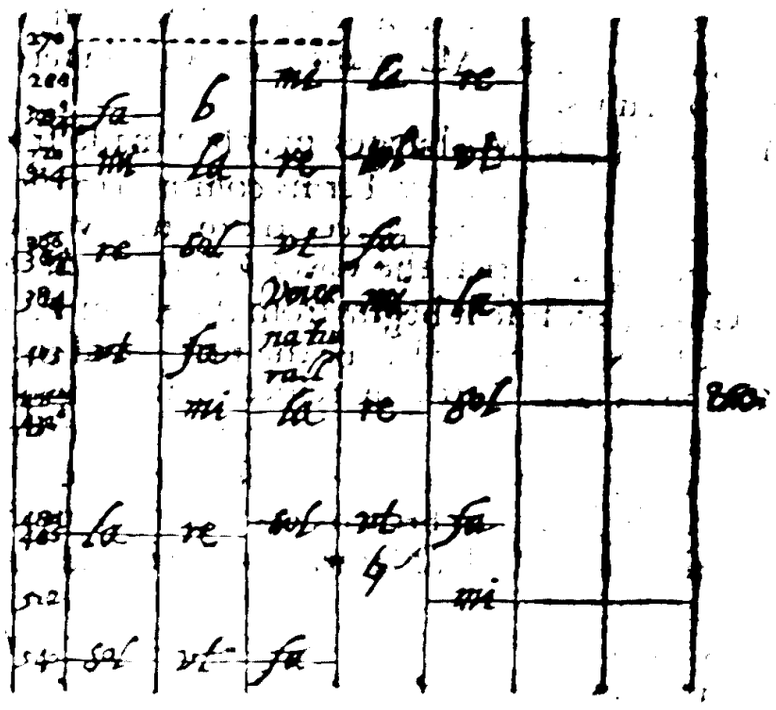
Thirdly, that there can be only two Kinds of an Artificiall Voyce, ♮ and ♭: because the space betwixt *A* and *C*, which is not divided in the Naturall voyce, can only bee divided by two Modes; so as that a Semitone be set in the first place, or the second.

Fourthly, for what reason these Notes, *ut, re, mi, fa, sol, la*, are againe repeated in those Artificiall Voyces: for Example, for, when wee ascend from *A* to *l*, insomuch as there are not other Notes, but *mi* and *fa*, to signifie a Semitone *majus*; it thence follows, that in *A*, *mi* is to be, *fa*, and in *b*, *fa*, and so in other places in order. Nor can you say, it had been more convenient to have invented other Notes; for they would have been superfluous, since they must have designed the same Intervalls, which are designed by those Notes in a Naturall voyce; besides they would have been inconvenious,

ous, because so great a multitude of Notes must have exceedingly troubled Musicians, as well in Reading, as singing of Tunes.

And lastly, how changes may bee made from one voyce to another, *viz.* by Tones common to two voyces: as also, that these voyces are mutually distant by a Fifth. [57] and that the voyce ♭ Flat, is the most Grave, because it begins from the Terme, which we have formerly proved to be the first, and therefore it is called ♭ Flat or Soft, in respect that by how much any Tone is the more Grave, by so much is it the more soft and remisse. For to the emission thereof is required the lesse spirit, or breath, as wee have more then once insinuated. And a Naturall voyce is and ought to be a mean, nor could it rightly be called *Naturall*, if the voyce were to be elevated, or depressed beyond Mediocrity, in the expression thereof. Finally, the voyce ♮, is called a Quadrate, or Sharp, because it is the most Antient, and the Opposite to ♭ Flat; as also, because [58] it divides an Eighth into a Trione and a Fifth (as [58], and is therefore lesse sweet than ♭ Soft.

Some perhaps will object, that the *Hand* is not sufficient to comprehend all the Changes of Degrees; for, as in it is shewn, how freely we may desert from a Naturall voyce, either to ♭ Soft, or to ♯; so also ought other collateral Orders to bee designed therein, such as are see in the Section Figure, that so it might have beene free for us also to desert from ♭ Soft, to the Naturall voyce, or to the other part, and so from ♯. Which is confirmed from hence, that Musicians in Practice frequently use such Intervals, which they explicate either by ♯ Sharp, or by ♭ Soft, which therefore separate from its proper Seat.



To this we return, that by this means might be made a progresse, *usq; ad infinitum*: but, in that *Hand*, ought to bee expressed the Changes of only one Tune; and that those are contained within three Orders, is demonstrated from hence, that in every Order only six Terms are contained, of which two are changed, when a change is made to the following Order, and so there remain therein only Four Termes of those, which were in the former; but if a Transition bee againe made to a Third Order, then will two Degrees of the four precedent ones bee changed, and so there will remain onely two of those which were in the former Order, which would lastly be taken away in the fourth Order, if the progresse should be continued unto it, as is visible in the Figure:

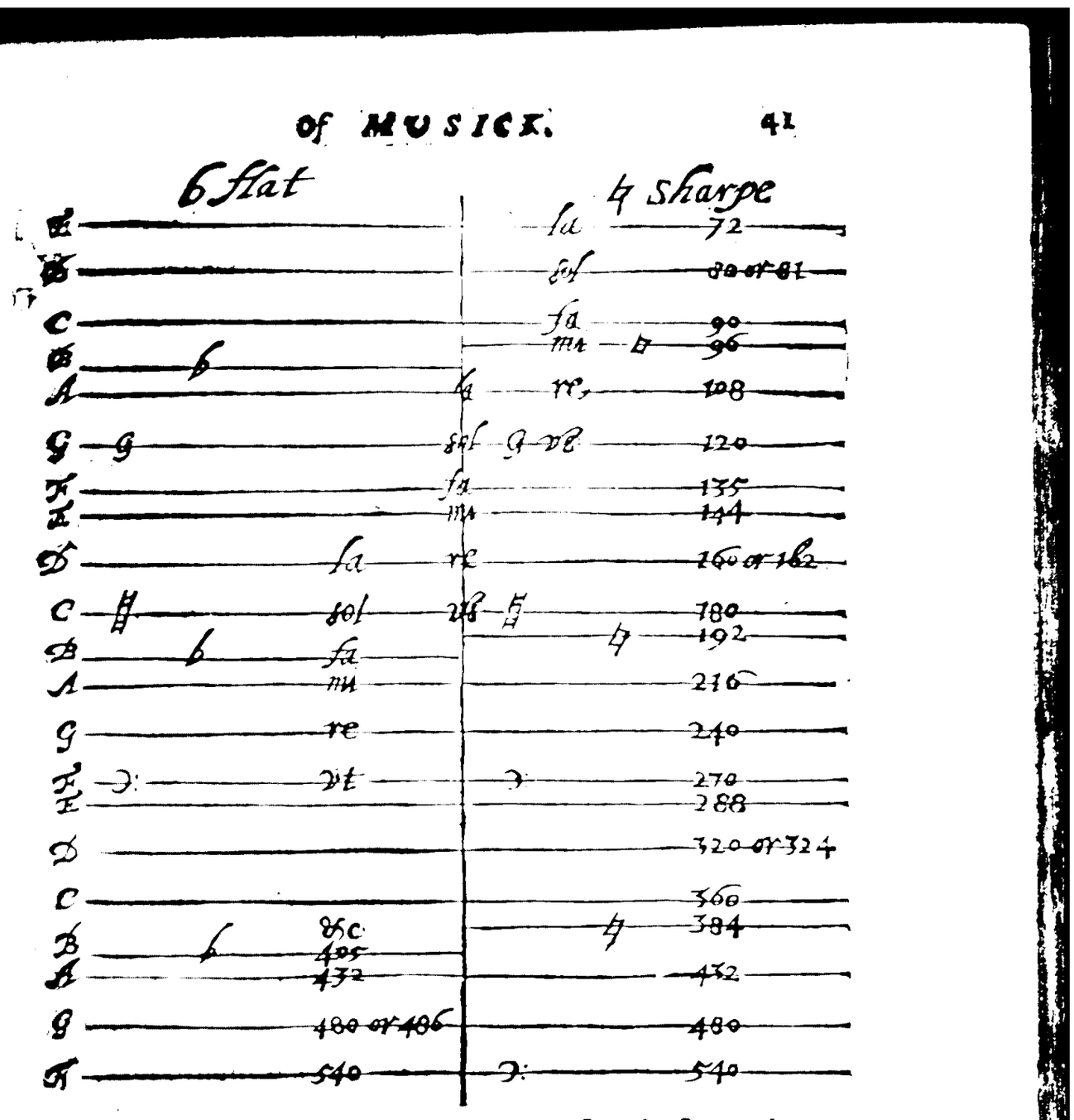
figure; whence it is moſt evident, that the Tune would not be the ſame it was in the beginning, ſince there ſelfe would remaine no Term unchanged. And what is added concerning the uſe of *Dieſes*; I ſay, that they doe not conſtitute whole Orders, as *b* Soft, or ♮, but conſiſt only in one Terme, which they elevate (as I conceive) by one Semitone *minimum*, all the other Terms of the Tune remaining unchanged; now the manner how, and the reaſon why this is done, I doe not at preſent ſo well remember, as to be able ſufficiently to explain; nor why, when only one Note is elevated above *la*, a *b* Soft is uſually affixed unto it: which I think may eaſily be deduced from Practice, if the Numbers of thoſe Degrees, in which they are uſed, and of voyces, which with them make Conſonances, bee ſubducted; and the matter I judge well worthy a ſerious Meditation.

Finally, here it may be objected, that ſix voyces, *ut*, *re*, *mi*, *fa*, *ſol*, *la*, are ſuperfluous, and only Four may ſuffice; ſince there are only three divers intervalls: by which way that any Muſicall Tune can be ſung, I deny not. But becauſe there is great difference betwixt the Terms Grave and Acute; and a Grave Term, as is formerly noted, is much the chiefeſt: therefore is it better and more commodious to uſe divers Notes, than the ſame towards an Acute part, and towards a Grave part.

This place requires us to explain the *Practica* of theſe Degrees, how Muſicall parts are conſtituted of them, and by what reaſon ordinary Muſick compoſed by practicall hands may be accommodated to what of the Theory hath been premiſed; that ſo all Conſonances and other its intervalls may bee exactly calculated. In order

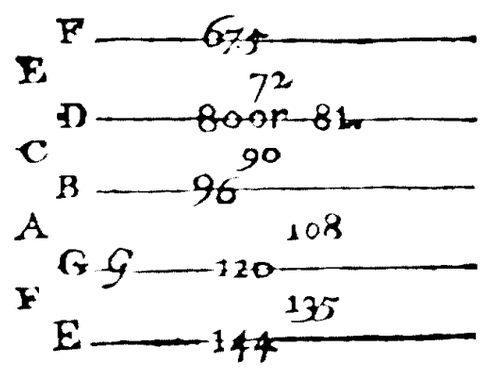
[illegible] of this, wee are to know, that [illegible] Cleifs in describing Musick betweene five lines, to which others also are added, if the Tones of the Tune bee further extended; and that these Lines are distant each from other, two Degrees, and therefore that betwixt each of them, one other is alwayes to bee understood, which is omitted for brevity & commodity sake. Againe, since all the Lines are equally distant each from other, but signifie unequall spaces: therefore are Two Markes invented, *b* and ♮, one whereof is set in that chord, which represents the Term *B fa* ♮ *mi*. Further, because one Tune doth frequently consist of many parts, which parts are separately described; it is not yet known, from those Marks, *b* and ♮, which of these many parts is superior, and which inferior: and therefore are there three other Marks found out, [illegible] the order whereof we have formerly observed [59]. Now that all these things may be the more manifest, wee have here placed this following Figure, in which wee have expressed all the Chords, and removed them each from other more or lesse, according to the greater or lesser spaces which they denote [60]; that so the proportion of Consonances might be presented together to the eye. Besides, wee have made this Figure double, that the Difference betwixt *b* and ♮, might be visible; nor can those Tones, which are to be sung by one, be described by the other, unlesse all the Tones of these be removed by a Fourth or Fifth, from their proper Seat, so that where stands the Term *F ut fa*, there is to be set *G sol re ut fa*.

Further

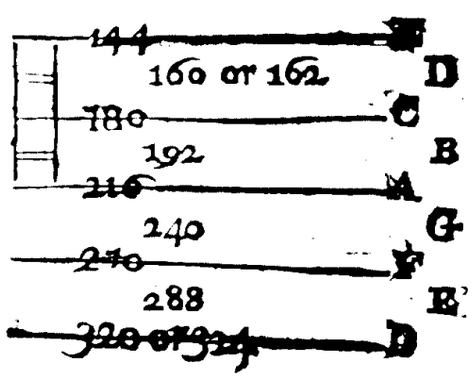


Further than this we are not to goe, for theſe ought to be the Terms, ſince they divide three Eights, within which all Conſonances are included, to which the Practice of Muſicians doth accord, for they hardly ever exceed this ſpace.

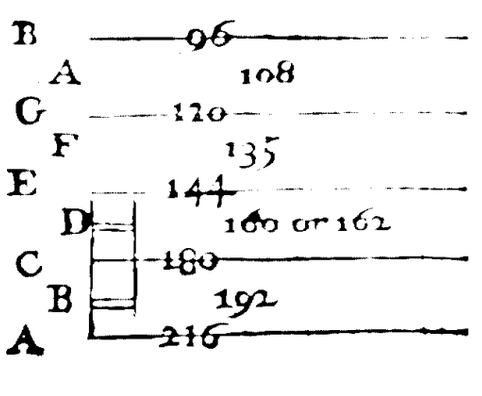
SUPERIUS.



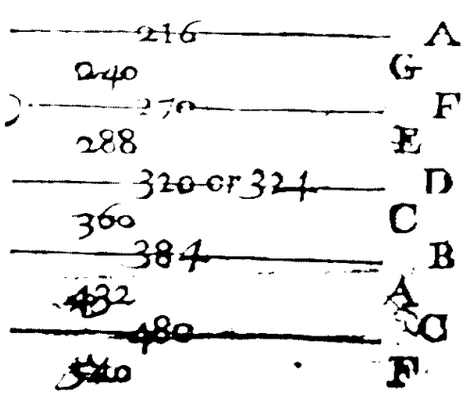
TENOR.



CONTRA TENOR.



BASSUS.



Now the uſe of thoſe Numbers is, to teach what proportion all the Notes hold among themſelves, ſuch as are contained in all the parts of one Tune: for the ſounds of theſe Notes hold the ſame proportion one to another, as the numbers appoſed on the ſame Chords. So as if the ſtring be divided into 540 equall parts, and the ſound thereof repreſent the moſt Grave Term F:

480

48a parts of the ſame ſtring will yield the ſound of the Term G; and ſo conſequently.

And here we have ordered 4 degrees of Parts, that it might appear, how much they ought to bee diſtant each from other; not that the Cliffs): , |=|, and G are not often ſet in other places, which is done according to the variety of Degrees, which are run over from each part: but becauſe this Mode ſeemes to bee the moſt Naturall, and is the moſt frequent.

Again, here have we ſet Numbers only in the Naturall Chords, and ſo long as they are not removed from their proper ſeat; but if Dieſes be found in ſome notes, or *b*, or ♮, which may remove them from their proper ſeats: then are thoſe to be explicated by other Numbers, whoſe quantity is to be deſumed from other Notes of other Parts, with which theſe kinds of Dieſes make a Conſonance.

CHAP. XI.

Of Diſſonances.

ALL other Intervalls, except thoſe of which wee have now ſpoken, are called Diſſonances; but we will treat of thoſe only, which are neceſſarily found in the newly explicated order of Tones, ſo as they cannot but be made uſe of and applyed.

Of theſe there are three kinds[61]: (1) Some are generated from Degrees only, and an Eighth: (2) Others from the difference which is betwixt a Tone *major* and *minor*, which we have denominated a Schiſm: and [61]

(3)others from the Difference, which is between a Tone
[62] *major*, and a Semitone *majus* [62].

In the *First Genus*, are contained Sevenths and Ninths, or Sixteenths, which are only Ninths compounded, as Ninths are nothing elſe but Degrees compounded of an Eighth, and Sevenths nothing but the reſidue of an Eighth, from which one Degree is detracted; whence it is manifeſt, that there are three divers Ninths, and three Sevenths, becauſe there are three kinds of Degrees; and all theſe conſiſt betwixt theſe Numbers
[63] [63]:

A	*Ninth maxim*	$\frac{8}{15}$	A	*Seventh major*	$\frac{8}{15}$
	Ninth major	$\frac{9}{16}$		*Seventh minor*	$\frac{9}{16}$
	Ninth minor	$\frac{15}{32}$		*Seventh minim*	$\frac{15}{32}$

Among Ninths, two are *majors*, which ariſe from two Tones, the Firſt from a *major*, the Second from a *minor*, for the diſtinction of which we have noted one Ninth maxim: on the contrary there are two Sevenths *minors*, for the ſame reaſon, and therefore we have called one Seventh minim.

Now, that theſe Diſſonances cannot be avoyded in ſounds ſucceſſively emitted, among divers parts is moſt clear: yet haply any one may enquire, why they ought not to be admitted in a voyce ſucceſſive of the ſame part equally with Degrees, ſince it is evident that ſome of them are explicated in leſſer Numbers than the Degrees themſelves, and therefore may ſeem to bee more
[64] grateſull to the Hearing than Degrees [64]. The ſolution of which Doubt doth depend on this, which we have
[65] before obſerved, that a voyce [65] doth require ſo much the

the more intenſion of the ſpirit or breath, by how much the more Acute it is, and therefore Degrees were invented, that they might be *Meanes*, betwixt the Termes of Conſonances, and that by them wee might the more eaſily aſcend from the Grave Terme of any Conſonance to the Acute of the ſame, or *vice verſa*, deſcend from the Acute to the Grave Term: which cannot be performed by Sevenths or Ninths; as is evident from hence, that the Termes of theſe are more diſtant each from other, than the Termes of Conſonances are, and therefore they would be emitted with a greater inequality of Contention.

In the *Second Genus* of Diſſonances do conſiſt a Third *minor*, and a Fifth Deficient by one Schiſme; as alſo a Fourth, and a Sixth *major* encreaſed by one Schiſme. For ſince (neceſſarily) there is one moveable Terme by the intervall of a Schiſme, in the whole Series of Degrees; it cannot be avoyded, but that, from thence, ſuch Diſſonances in relation, *i.e. in voce ſucceſſivè emiſſa a diverſis vocibus*, will bee generated: And that more then theſe now named cannot ariſe from thence, may bee proved by induction [66]. Theſe conſiſt in theſe Numbers [67]: [66] [67]

$$A\left\{\begin{array}{ll} \textit{Third minor defective} & \frac{27}{32} \\ \textit{Fifth defective by one Schiſm} & \frac{27}{40} \\ \textit{Fourth increaſed by one Schiſm} & \frac{20}{27} \\ \textit{Sixth major increaſed by a Schiſm} & \frac{48}{81}\,\frac{16}{27} \end{array}\right.$$

[68] Or thus [68]:

A
Third minor defective	G ad D.	480, 405.
by a Schisme	d ad D.	384, 324.
Fifth defective by one Schisme	G ad D.	480, 324.
Fourth encreased by one Schisme	D ad G.	324, 240.
Sixth major encreased by a Schisme	b ad G.	405, 240.
	D ad d.	324, 192.

But so great are these Numbers, that such intervalls cannot be collected of themselves; but, as we have formerly noted, because the intervall of a Schisme is so small, as it can hardly bee discerned by the ears, therefore doe they borrow sweetnesse of those Consonances, to which they are neares. Nor doe the Terms of Consonances so consist *in indivisibili*, as that if one of them be a little changed, all the sweetnesse of the Consonance must instantly be lost: and this can only be the reason, why Dissonances of this *second Genus* may be, in a voice successive of the same part, admitted in place of Consonances, from which they are divided.

In the *Third Genus* are contained, a Tritone, and a Fifth false; for in this, a Semitone *majus* is accounted for a Tone *major*; but in a Tritone, the Contrary: and they [69] are explicated by these numbers [69]:

Tritone $\frac{32}{45}$ *Fifth false* $\frac{45}{64}$

Or

Or thus [70]: [70]

Tritone { F *ad* pl. 540, 384.
b *ad* E. 405, 288.

Fifth falſe { pl *ad* F. 384, 270.
E *ad* b. 288, 202; *vel* 576, 405.

Which Numbers are alſo too great to explicate any intervall which may not be ingrate to the ears; nor have they any Conſonances very near, from which they may borrow ſweetneſſe, as the Precedent ones have. Hence comes it, that theſe laſt Diſſonances ought to be avoided in relation; at leaſt, when ſlow and ſoft Muſick is made, and not diminute; for in very diminute Muſick and ſuch as is ſung ſwiftly, the hearing is too much imployed to take notice of the defects of ſuch Diſſonances: which defect is much more evident from hence, that they are near to a Fifth, with which the Hearing therefore compares them, and, from the precipuous ſweetneſſe of this, doth the more clearly diſcern the imperfection of thoſe.

Here we ſhall end our explication of all the Affections of a Sound; having firſt only taken notice, in order to the probation of what we formerly ſaid, that all the Variety of ſounds, as to Grave and Acute, doth ariſe in Muſick onely from theſe Numbers 2, 3, and 5. We ſay that all numbers, by which as well Degrees, as Diſſonances are explicated, are compoſed of thoſe three, and by them, diviſion being made, may at length be reſolved even to an unity.

CHAP.

CHAP. XII.

Of the reaſon of compoſing.

FRom the Premiſes it followes, that we may, without any great errour or ſoloeciſm, compoſe Muſick, if we obſerve theſe 3 axioms.

1. That all ſounds which are emitted together, may be diſtant each from other, in any Conſonance, except a Fourth, which loweſt ought not to be heard, *i.e.* againſt a Baſſe.

2. That the ſame voice be moved ſucceſſively, only by Degrees, or Conſonances.

3. Laſtly, That we admit not a Tritone, or Fifth falſe, no not ſo much as in relation.

But, for the greater Elegancy and Concinnity, we are to note theſe following Rules.

1. That wee begin from ſome one of the moſt perfect Conſonances; for, ſo is raiſed a greater attention, than if ſome jejune and frigid Conſonance led up the Van: or elſe, moſt gratefully, from a pauſe or ſilence of one voyce; for when, immediately upon the ſilence of one voyce, which began the Tune, another unexpected one Firſt invades the ears, the novelty thereof doth by a kind of potent charm, conjure us to attention. Now, concerning a Pauſe we have been hitherto ſilent, becauſe of it ſelf a Pauſe is nothing, but onely induceth a certain novity and variety, while the voyce, which was ſilent, doth againe begin to ſing.

2. That two Eights, or two Fifths never immediately

ately ſucceed each other. The reaſon why that is prohibited more expreſly in theſe Conſonances than in others, is becauſe theſe are the moſt perfect, and therefore when one of them is heard, then is the Hearing therewith fully ſatisfied, and unleſſe the attention bee preſently removed from that to another Conſonance, it is wholly occupied by the pleaſantneſſe thereof, ſo that it can little regard the variety, and the (in ſome ſort) frigid Symphony of the Tune; which happens not in Thirds and other Conſonances, no though they be reiterated, for in all others the attention is ſtill kept up, and a deſire encreaſed of expecting a more perfect Conſonance.

3. That ſo much as poſſible, the parts goe on in contrary motions, in order to the greater variety: for then both the motion of every voice is diſtinguiſhed from the adverſe voice, and Conſonances are diſtinguiſhed from other Conſonances near them. Alſo that all the voyces be moved oftner by Degrees, than by leaps.

4 That, when we would advance from any leſſe perfect to a more perfect Conſonance, wee alwayes deflect to one that is near, rather than to one that is remote; for example, from a Sixth *major* to an Eighth, from a Sixth *minor* to a Fifth, *&c.* underſtanding the ſame alſo of an Uniſon and the moſt perfect Conſonances. Now, the reaſon why this method is to bee obſerved in progreſſion from imperfect to perfect Conſonances, rather than *e contra*, from perfect to imperfect; is, becauſe, when we heare an imperfect Conſonance, the eares are induced to expect a more perfect one, wherein they may receive more ſatisfaction, and to this expectation are they carryed by a certain naturall violence: and there-

fore ought a more vicine, than a remote Conſonance rather to be ſet, that being what the Hearing deſires. But, on the contrary, when a perfect Conſonance is heard, we expect no imperfect one. Yet this Rule is ſubject to frequent variation, nor can we now call to mind, from what to what Conſonances in particular, and by what motions wee ought to pervene: all theſe depend on experience, and the practice of Muſicians; which being known, we conceive it no difficulty to deduce the reaſons and proportions of all from this our Theory of Muſick: and I have formerly deduced many of them, but my peregrinations have worn them out of both my Papers and Memory.

5. That, in the end or cloſe of each Tune, the eares be ſo fully ſatisfied, as they expect no more, but perceive the Tune to be perfect: which is moſt conveniently effected by ſome Orders of Tones alwayes ending in a moſt perfect Conſonance, which Orders Muſicians call Cadences, all the Species of which Cadences have been copiouſly enumerated by *Zarlinus*. Who hath Generall Tables or Schems alſo, wherein are deſcribed what Conſonances in particular ought to ſucceed each other through a whole Tune; of all which hee hath given ſome reaſons, but we believe that more and more plauſible ones, may be deduced from our Fundaments.

6. And laſtly, that the whole Tune together, and every voyce ſeperately be included within certain limits, which are called Modes, of which anon.

All theſe Rules are to bee exactly obſerved in the Counter-poynt of only two, or more voices; but not in a Diminute, nor any way varied: for in Tunes very Diminute, and (as they call them) Figurate, many of them

them are remitted. Which that we briefly explicate, wee are concerned first to treat of the foure Parts, or Voices used in Tunes; for though in some are found more, in some fewer Symphonies: yet that seems to bee the most perfect and most usuall Symphony, which is composed of four Voices.

The First and most Grave of all these Voices, is that which Musicians call *Bassus*. This is the chiefe, and ought principally to fill the ears, because all other Voices carry the chiefest respect to the *Basse*, the reason whereof we have formerly declared. Now, this Voice is wont to move on not onely by Degrees, but also *per Saltus*; the reason is, because they were invented to ease that trouble, which would arise from the inequality of the Terms of one Consonance, if one should immediatly bee expressed upon the neck of another; since the more Acute doth strike the eare much more forcibly than the Grave. For this trouble is lesse in a *Basse*, than in other parts; in respect that it is the most Grave, and therefore requires lesse strength of the spirit or breath to its effusion, than any other. Besides, since all other Voices hold a respect to the *Basse*, as the principall; it ought to strike the ears more sensibly, that it may bee heard more distinctly: which is effected, when it moves on *per Saltus*, *i. e.* by the Terms of lesser Consonances immediately, rather than when it moves on by Degrees.

The Second, being the next to the *Basse*, they call *Tenor*; this being also, in its kind, the chiefest, because it containes the Subject of the whole Modulation, and is comparatively the Nerve, which extended through the body of the Tune, doth sustain and conjoyn all the rest of its Members. And therefore it is wont, so much as

poſſible, to move on by Degrees; that ſo its parts may be the more united, and the Notes of it may be the more eaſily diſtinguiſhed from the Notes of other Voices.

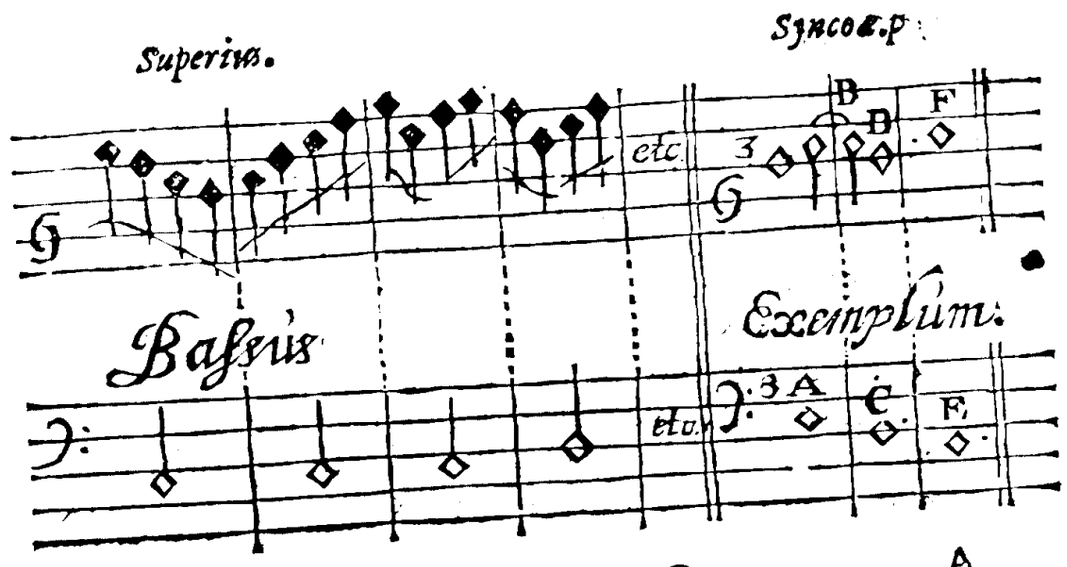
To the *Tenor* is oppoſed the *Contra-tenor*; nor is it uſed in Muſick for any other reaſon but becauſe, by progreſſing to contrary motions it may occaſion Variety, and ſo Delight. It is wont, as the *Baſſe*, to move on by leaps; but not for the ſame reaſons: for this is done only for convenience and variety; for it conſiſts betweene two voices, which move on by Degrees. Practiſers ſo compoſe their Tunes ſometimes, that they deſcend below a Tenor; but this is of ſmall moment, nor doth it ſeem at any time to adfer any novity, unleſſe in imitation, conſequence, and the like artificiall counter-poynts.

Superius is the moſt Acute voice, and is oppoſed to *Baſſus*, ſo that by contrary motions they frequently occur each to other. This voice ought chiefly to incede by Degrees; becauſe, ſince it is moſt Acute, the difference of Terms in this would cauſe greater trouble and difficulty, if thoſe Terms, which it would ſucceſſively emit, were at too great diſtance each from other. And it is wont to be moved the ſwifteſt of all others in Diminute Muſick: as the *Counter-Baſſe* moſt ſlowly: the reaſons whereof are deduceable from our precedent diſcourſe; for a more remiſſe ſound ſtrikes the Ears more ſlowly, and therefore the Hearing cannot endure ſo ſwift a mutation therein, in reſpect it would not have leaſure to hear all the ſingle Tones diſtinctly.

Theſe things thus explained, we are not to omit, that in theſe Tunes Diſſonances are frequently uſed inſtead of Conſonances; which is effected two wayes, *viz.* by Diminution

Diminution, or Syncope.

1. *Diminution* is when againſt one Note of one part, are ſet 2. or 4. or more in another; in which this order ought to be kept, that the Firſt make a Conſonance with a Note of another part, but the Second, if it be only one Degree diſtant from the former, may make a Diſſonance, and alſo be, by a Tritone, or Fifth fals, diſtant from another part, becauſe then it ſeems there ſet only by accident: and as a way, by which wee may come from a Firſt Note to a Third, with which that Firſt Note ought to make a Conſonance, as alſo doth the Note of the oppoſite part. But, if that Second Note incede *per Saltus*, *i. e.* bee diſtant by the intervall of one Conſonance from the Firſt; then ought it to make a Conſonance alſo with the oppoſite part: for the former reaſon ceaſeth. But then a Third Note may make a Diſſonance if it be moved by Degrees; of which let this be an Example.



A *Syncopa* is, when the end of one Note in one voice is heard at the ſame time with the beginning of one other Note of an adverſs part; as may bee ſeene in the Example ſet, where the laſt time of the Note *B*, is diſſonant with the beginning of the Note *C*, which is therefore brought in, becauſe there is yet remaining in the eares the recordation of the Note *A*, with which it made a Conſonance; and ſo *B* bears it ſelfe to *C*, only as a Relative voyce, in which the Diſſonances are carryed through: yea, the Variety of theſe doth cauſe, that the Conſonances, among which they are ſet, are heard more diſtinctly, and alſo excite the more conſtant attention. For, when the Diſſonance *B C* is heard, the expectation of the eare is encreaſed, and the judgement of the ſweetneſſe of the Symphony ſomewhat ſuſpended, untill the Tune ſhall arrive at the Note *D*, in which it more ſatisfies the Hearing; and yet more perfectly in the Note *E*, with which, after the end of the Note *D*, hath kept up the attention, the Note *F*, inſtantly ſupervenient doth make an exquiſite Conſonance, for it is an Eighth [71]. And, indeed, therefore are theſe Conſonances uſed in Cadences; becauſe what hath been the longer expected, doth the more pleaſe when it comes: and therefore the ſound, after a Diſſonance heard, doth better acquieſce in a moſt perfect Conſonance, or Uniſon. But heere Degrees are to be ſet betwixt Diſſonances: for whatever is not a Conſonance, ought to be accounted a Diſſonance.

[71]

Moreover, wee are to obſerve, that the Hearing is more ſatisfied in the end by a Eighth, than by a Fifth, and beſt of all by an Uniſon; not becauſe a Fifth is not gratefull to the eare, as to the reaſon of Conſonance: but

but becauſe in the end we are to regard Quiet, which is found greater in thoſe ſounds, betwixt which is leſſe difference, or none at all, as in a Uniſon. Now this Quiet, or Cadence is delectable not only in the end: but alſo in the midle the avoidance of this Cadence introduceth no ſmall delight; namely, when one part ſeems willing to quieſce, and another proceeds on. And this is a kinde of Figure in Muſick, ſuch as are Rhetoricall *Figures* in Oration, of which ſort are *Conſequence*, *Imitation*, *&c.* which are effected, when either two parts ſucceſſively, *i. e.* at divers times, ſing wholly the ſame, or a quite Contrary, which at laſt they are wont to doe. And truely this, in certain parts of a Tune, doth ſometimes much advantage Muſick; but as for thoſe artificiall *Counter-poynts*, as they call them; in ſuch Compoſures where that Artifice is obſerved perpetually from the beginning to the end: we conceive, they may belong not more to Muſick, than *Acroſticks*, or retrograde Verſes to *Poeſie*, which was invented to charm the mind into reſpective paſſions, as well as Muſick.

CHAP. XIII.

Of Modes.

FRequent it is among Practitioners to treat of theſe Modes, and what they are, all well know; therefore would it be ſuperfluous here to inſiſt thereon: wee ſhall obſerve only, that they have their originall from hence, that an Eighth is not divided into equall Degrees, for one while a Tone, another while a Semitone

tone is found therein: and beſides, from a Fifth, becauſe that of all others is moſt acceptable to the eare, and every Tune ſeemes to bee compoſed for the ſake of this alone: for an Eighth can be divided into Degrees,
[72] onely ſeven different wayes [72], every one of which
[73] may bee againe divided by a Fifth two wayes [73], ex-
[74] cept Two [74]; in one of which is found a Fifth falſe
[75] in place of a Fifth [75], whence there ariſeth onely twelve Modes, of which foure are leſſe elegant, for this
[76] cauſe, that a Tritone is found in their Fifths [76], ſo as they cannot, from a Fifth principall, and for whoſe ſake the whole Tune ſeems compoſed, aſcend or deſcend by Degrees, but of neceſſity there muſt occur a falſe Relation of a Tritone, or a Fifth falſe.

In every Mode, are three principall Termes, from which all Tunes ought to bee begun, and moſt chiefly
[77] concluded [77], as all Muſicians know: and they are called Modes as well from hence, that they reſtrain the Tune, leaſt the parts of it ramble beyond mediocrity to exceſſe; as from hence chiefly, becauſe they are apt to containe various Tunes, which may diverſly affect the minde according to the variety of Modes; of which many things have been ſayd by Practiſers, taught onely by experience, the reaſons of all which may be deduced from our precedent diſcourſe: for, certaine it is, that in ſome many Ditones, or Thirds *minors*, and in places more or leſſe principall, are found, from which almoſt all the variety of Muſick doth ariſe, as hath beene formerly proved. Again, as much may be ſayd of Degrees themſelves; for a Tone *major* is the Firſt, and comes neareſt to Conſonances, and is *per ſe* generated from the
[78] Diviſion of a Ditone; but all others *per Accidens* [78], from

from which and the like, many things concerning the nature of Moods might bee deduced, if a *Compendium* would permit. And heere it should follow, that wee should discourse of all the motions of the minde, which may bee excited by Musick, and in a singular Treatise shew, by what Degrees, Consonances, Times, *&c.* those motions ought to bee excited: but I should bee unconstant to my purpose of writing an Epitome.

I now discover Land, hasten a shoare, and omit many things for brevity, many by oblivion, but more by ignorance. However, I suffer this issue of my braine, so inform, and lately brought forth rude as a Bears Cub, to venture abroad into your presence: that it may remain as a Monument of our Familiarity, and a most certain memoriall of my love of you: yet, if you please, upon this condition, that, being confined to the secrete of your Closet, it bee not exposed to the judicature of others, who may not (as I trust you will) avert their benevolous eyes from the maimed, and defective parts of this Exercise, upon those others, in which I deny not but I have expressed some Lineaments of my Ingenie to the life; nor would they know that this *Compendium* was composed for your sake alone, by one who could not obtain Privacy in an Army, nor leasure in a Throng of other Cares and Affairs.

CONTENTS.

CHAP.

FINIS.

ANIMADVERSIONS
UPON THE
Muſick-Compendium
OF
RENAT. DES-CARTES.

LONDON,

Printed by *Thomas Harper*, for *Humphrey Moſeley*, and are to be ſold at his Shop at the Sign of the *Princes Armes* in S. *Pauls* Church-Yard. 1653.

Animadversions upon the Musick-Compendium of R. Des-Cartes.

In these subsequent *Animadversions, brevitatis gratia,*

Roote, or Side		√
Addition, or more		+
Subduction, or lesse		−
Æqualitie		=
Aggregate, or Sume		Z
Excesse, or Difference		X.
Lower, or Graver		▽
Lower, or Graver Terme		▽̄
Higher, or Acuter		△
Higher, or Acuter Terme		△̄
Ration		R
Æquality of Ration, or proportionall		::
Continued Proportion		∴
Multiplyer, or multiplyed by		m
Diviſor, or divided by		⊏
Product		⊏
Quotient		⊏
Power		⊏
Logarithme		⊏

I Characterize thus:

And, *diſtinctionis cauſa*, I denominate the firſt Note or Term of any Conſonance, or other Muſicall Intervall, an *Uniſon*; and the other, according to its difference, in ſound, from the former.

[1] *Audible Differences are as viſible Rations*: For Sounds cannot bee diſtinguiſhed, or their Differences known otherwiſe than by their mutuall habitude. underſtand me as thus: The *Sounds* of ſtrings are according to their *Rations*, not viſible *Differences*: for Example, as theſe three *Chords* have an *equality* of *Rations*: (for *a.b* :: *b.c.*) ſo their *Sounds* (an *Uniſon*, *Eighth*, and *Fifteenth*) have an *equality* of *Differences*. (For $1+\overset{*}{7}=8$, and $8+\overset{*}{7}=15$.) And as theſe three *Chords* have an *inequality* of *Rations*: (though an equality of Differences viſible; for $d+\overset{*}{g}=e$, and $e+\overset{*}{g}=f$.) ſo their *Sounds* (an *Uniſon*, *Fifth*, and *Eighth*) have an *inequality* of *Differences audible*. For as the Ration of *d* to *e*, is $\frac{2}{3}$: (and $\frac{2}{3}$ is a Fifth, by Fig. firſt, p.10.) ſo the difference of an Uniſon and a Fifth is a Fifth. ($1+\overset{*}{4}=5$.) and as $\frac{3}{4}$ of *e* to *f* is $\frac{3}{4}$: (and $\frac{3}{4}$ is a Fourth by Fig. firſt, p. 10.) ſo the difference of a Fifth and an Eighth is a Fourth. ($5+\overset{*}{3}=8$.) And (therefore) Sounds, thus numbred, are as it were imperfect (becauſe not equally diſtant) audible Indices, or Logarithms of their Chords. Here the Reader may obſerve that for the Difference of an *Eighth*, I have added

ded only *ſeven*; of a *Fifth*, *four*; and of a *Fourth*, *three*: and the reaſon is, becauſe the excluſive account is alwayes one leſſe than the incluſive, as is made viſible *Animad.* 8.

[2] *Viz. Arithmeticall.* Whereof on ſtrings are two ſorts; one *audible*, the other *viſible*; but, as to their meaſure, the *Laſt* only is properly called *Arithmeticall*; the *firſt Rationall*, or *Geometricall.*

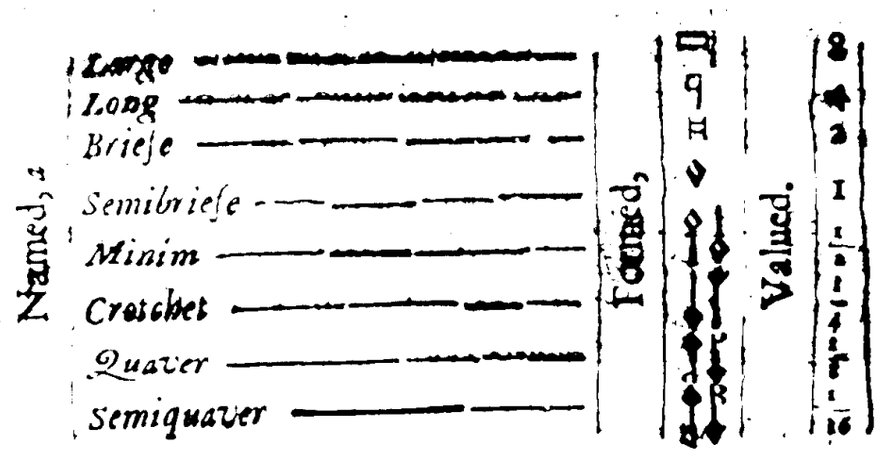
[3] Note there are in *Sounds* two *Proportions*, and *Progreſſions*, as well as in Lines and Numbers; *viz.* the *Arithmeticall*, as Second, Third, and Fourth: for $2-1=3-2=1$: and the *Geometricall*, as Second, Third, and Fifth: for 1. 2. : : 2. 4. And note alſo, as was ſayd before Animad. Firſt: That when *Strings* are *audibly* in an *Arithmeticall* proportion, or progreſſion, they then are *viſibly* in a *Geometricall*; whence I infer that Chords, as to *Sounds*, ought to be Geometrically divided, not Arithmetically; becauſe, ſo divided, the ſence of hearing has not ſo much to adverriſe; the audible Differences being alwayes equall, *&c.* whereof more, after Anim. 78, P. 1.

[4] $\surd 8 = 2{\cdot}828+$, therefore is $\begin{cases} a\,b = 0{\cdot}828+ \\ b\,c = 1{\cdot}172- \end{cases}$

[5] *Viz.* 0·8.

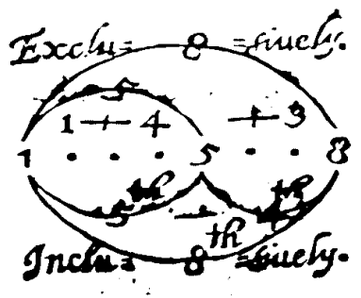
[6] *Viz.* 1·2.

[7] The Notes, or Markes of *Time*, in Muſick are Named, thus



But note theſe Markes are found otherwiſe valued ſometimes; as when a Large doth comprehend three Longs, a Long three Brieſes, *&c.* according to their ſeverall Moods; or Moods, Times, and Prolations: For ſatisfaction wherein, as in all things elſe practicall in Muſick, not neceſſary to be known, as to the underſtanding of this *Compendium*, the Reader is referred to *Harmonicon Merſenni*, *Muſurgia Kertheri*, Morleys Introduction, *&c.*

[8] That is, is *Four* or *Seven Notes* higher: For the *Fifth* is the *Fourth* from the Firſt, and the *Eight* is the *Seventh*, *&c.* The knowledge of which Notes, together with all other *Conſonances*, and *Muſicall Intervalls* (ſome few excepted, not now in uſe,) may bee, without difficulty, obtained by inſpection on the firſt Figure following.



Whereof

Whereof the *Space* from the *Bridge* to the *Nutt*, is understood to be divided into 540, or 10·000 equall parts: the Number of which parts (accounting from the Bridge) to each actuall division of the foure Chords, or Strings, numbred at the Bridge 1, 2, 3, 4; is to be found on the Right hand. The first (B 0) presents you all the Intervalls under an *Eighth*; and their proportions, names, and differences by paralell entrance thence towards the Right hand. and is thus to be read: *viz.* B 0 [540, or 10·000], is to B 1 [518·4, or 9·600], as 25, to 24: as an *Unison*, to its Acuter *Semitone minus*: B 0 [540, or 10·000]. B 2 [506·25, or 9·375] :: 16.15 :: *Unison.* △ *Sem. major*: B 21 [270, or 5·000]. B 20 [281·25, or 5·208 $\frac{1}{3}$] :: 24.25 :: *Unison.* ▽ *Sem. minor*: B 21 [270, or 5·000]. B 19 [288, or 5·333 $\frac{1}{3}$] :: 15.16 :: *Unison.* ▽ *Semit. major*: The *Habitude*, or Proportion of B 1, to B 2; or of B 2, to B 1: or the difference of a Semitone *minor*, and *major*; or of a Seventh *major*, and Semi-Eighth; is a *Diesis minor*, *&c.* Hence it appeareth that B 0, if struck, when stop'd at 1, doth sound a Semitone *minor* more acute, than it doth, if struck, when unstop'd or open: and that a Semitone *minor* (as 01) is equall to $\frac{1}{25}$ of the $\frac{*}{▽}$, and is substracted from it; and $\frac{1}{11}$ of the $\frac{△}{*}$, and is added to it. And the like (*mutatis mutandis*) in all the Rest.

The *Second Chord* (VF) is divided according to b flat: the *Third* (LF) according to ♮ shape: both, from F to F, as in the Scale, P. 41. And the *Fourth* (W A,) as these, and the like *Instruments*, are usually fretted.

Thus having all the *Intervalls* under an *Eighth*, those above are easily known: for they are all compounded either of one, or more Eighths only; as the *Fifteenth*, Two & twentith, Nine and twentith, &c. or else of one, or more Eighths, and some one of these. And (therfore) as B 0 was divided, to make the first seven Notes after, or above the *Unison*, so is B 21 understood be divided, to make the seven next after, or above the *Diapason*, *&c. ad infinitum*.

Their Proportions, Denominations,

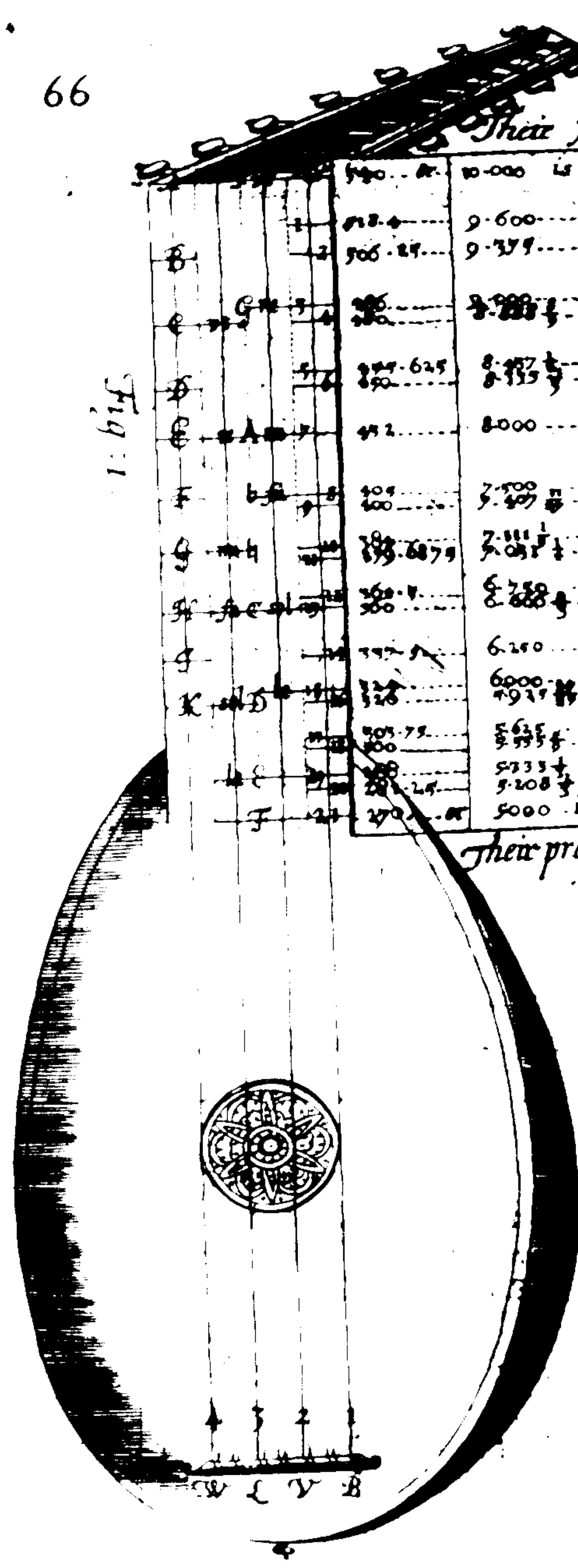


Fig. 1

540 &c.	10.000 is to	as	to	as an Unison, to its acute
518.4	9.600	25	24	Semitone minor; or Diesis major
506.25	9.375	16	15	Semitone major; or Degree minor
486 / 480	9.000 / 8.888 8/9	10 / 9	9 / 8	Tone or second minor; or degree major / Tone or second major; or degree minor
455.625 / 450	8.437 1/2 / 8.333 1/3	32 / 6	27 / 5	Third minor — Schisme / Third minor; or Semiditone
432	8.000	5	4	Third major; or Ditone
405 / 400	7.500 / 7.407 11/27	4 / 27	3 / 20	Fourth, or Tessaron / Fourth + Schisme
384 / 379.6875	7.111 1/9 / 7.031 1/4	45 / 64	32 / 45	Tritone / Semifyfth
360 / 360	6.750 / 6.666 2/3	40 / 3	27 / 2	Fifth — Schisme / Fifth; or Diapente
337.5	6.250	8	5	Sixth minor; or diapente + Semit: major
324 / 320	6000 / 5.925 25/27	5 / 27	3 / 16	Sixth major; or diapente + Tone minor / Diapente + Tone major
303.75 / 300	5.625 / 5.555 5/9	16 / 9	9 / 5	Seventh minor; or Diapason — Tone major / Seventh minor; or Diapason — Tone minor
288 / 281.25	5.333 1/3 / 5.208 1/3	15 / 48	8 / 25	Seventh major; or Diapason — semitone major / Semi-eighth
270 &c.	5000 is to	2	1	Eight, or Diapason

Their propor:

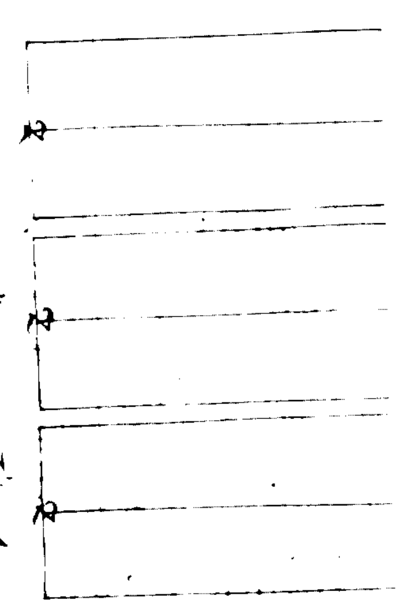


Fig. 2.

Fig. 3.

Fig. 4.

Differences.

1	2	Eighth, or Diapason	
			Semitone minor, Diesis major, or Chromatica.
25	48	Semi-Eighth	
8	15	Seventh major	Diesis minor, or Enharmonica. as 128 to 125. Δ
			Semitone minor.
5	9	Seventh minor	
9	16	Seventh minime	[illegible] as 81 to 80 Δ. as [illegible] to [illegible] ∇.
			Semitone, or Limma Pythag. as 243 to 256. ∇
16	27	Fifth + Second major	
3	5	Sixth major	Schisme, or Comma [illegible]
			Semitone minor.
5	8	Sixth minor	
			Semitone major.
2	3	Fifth	
27	40	Fifth — Schisme	Schisme.
			Semitone minor.
45	64	Semififth	
32	45	Tritone	Comma minus i.e. as 2048 to 2025. Δ
			Semitone minor.
20	27	Fourth + Schisme	Schisme.
3	4	Fourth	
			Semitone maior.
4	5	Third major	
			Semitone minor } Semitone med. as 135·128. Δ
5	6	Third minor	
27	32	Third minor — Schisme	Schisme.
			Semitone pythag. as 256 to 243. Δ
8	9	Tone major	
9	10	Second minor	Schisme.
			Semitone minor. } Semitone major as 27 to 25. Δ
15	16	Semitone major	
24	25	Semitone minor	Diesis [illegible]
			Semitone minor.
45	to	as unisone, to its graver.	

tions, Denominations, Differences.

540	A	10.000
510.3	B	9.450
482.2	C	8.920
455.7	D	8.438
430.6	E	7.974
406.9 / 405.9	F	7.535 / 7.517
384.5	G	7.120
363.3	H	6.728
343.3 / 339.3	G	6.358 / 6.282
324.4	K	6.008
306.6	L	5.677
289.7	M	5.369
273.7 / 269.1	N	5.069 / 4.984
258.7	O	4.790
244.4	P	4.527
231.0	Q	4.278
218.3	R	4.042
206.3	S	3.820

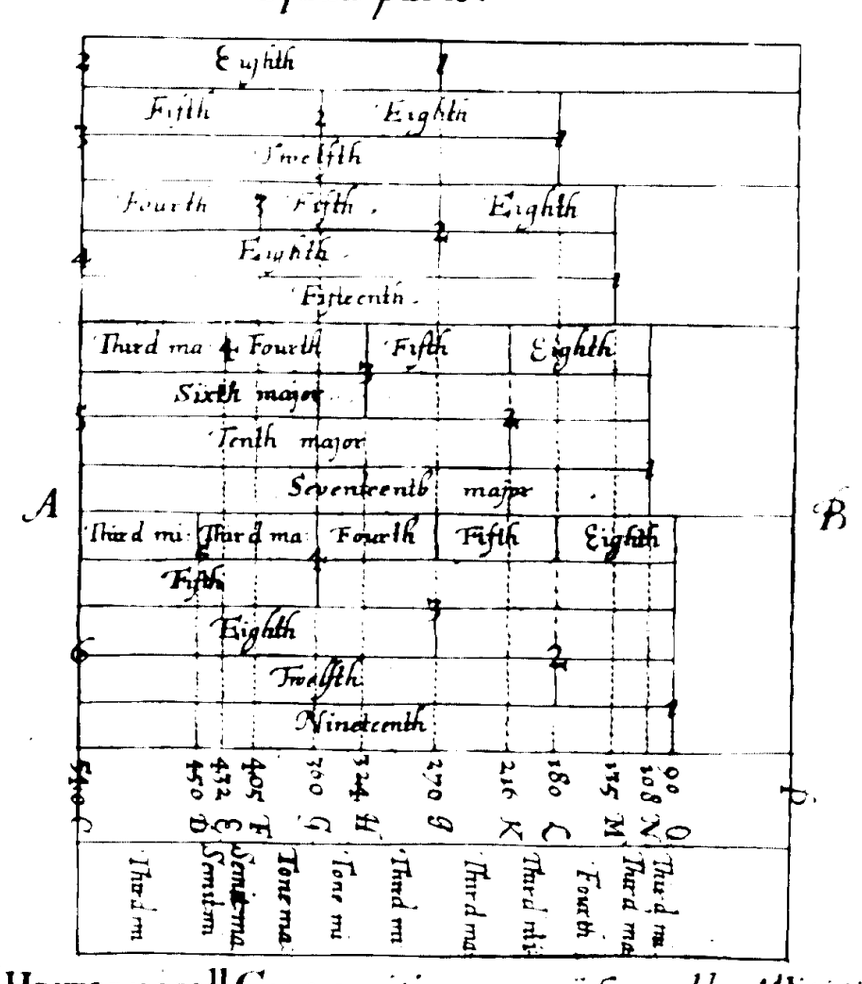
540	A	10.000
509.7	B	9.439
481.1	C	8.909
454.1	D	8.409
428.6	E	7.937
404.5	F	7.492
381.8	G	7.071
360.4	H	6.674
340.2	I	6.300
321.1	K	5.946
303.1	L	5.612
286.1	M	5.297
270.	N	5.000

540	A	10.000
509.2	B	9.429
480.1	C	8.898
452.7	D	8.384
426.9	E	7.905
402.5	F	7.454
379.6	G	7.029
357.9	H	6.628
337.5	I	6.250
318.2	K	5.893
300.1	L	5.557
282.9	M	5.244
266.8	N	4.941
251.6	O	4.659
237.2	P	4.393
223.7	Q	4.142

Animadversions upon the

[9] *Yett, in his Second figure p.13, y^e* Author *sets downe some* Consonances *with greater* Differences; *and page.14 he dichotomiseth A B in to eight parts for the* Consonances, *as into 16 for both* Tones.

[10] *But more clearly this fig: following, where the* Space, A B *is actually and distinctly diuided into 2,3, 4,5, & 6, equall parts.*



[11]. *All* Harmonicall Compositions *are performed by Actio of their*

their *Rations*, and Divisions by Subduction: *viz.*

Addition, by a Multiplication of the *like* Terms, or Collaterally thus =:

Substraction by a Multiplication of the *unlike* Terms, or obliquely thus X:

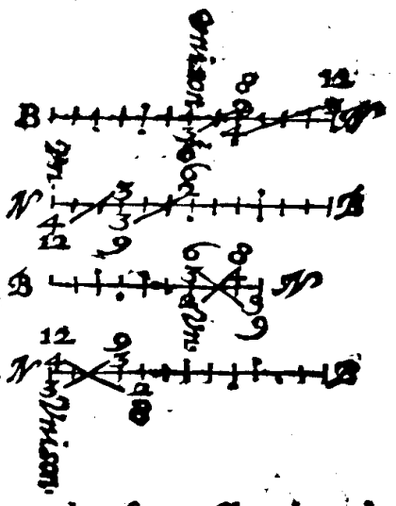
For Example.

$\frac{1}{4} + \frac{1}{3} = \frac{4}{12} = \frac{1}{1}$. i.e. $\nabla 4 + \nabla 5 = \nabla 8$.

$\frac{4}{3} + \frac{1}{2} = \frac{12}{6} = \frac{2}{1}$. i.e. $\Delta 4 + \Delta 5 = \Delta 8$.

$\frac{2}{3} - \frac{3}{4} = \frac{8}{9}$ i.e. $\nabla 5 - \nabla 4 = \nabla 2$ Major.

$\frac{2}{2} - \frac{4}{5} = \frac{9}{5}$. i.e. $\Delta 5 - \Delta 4 = \Delta 2$ Major.



as is visible from the divisions on the *foure Chordes* adjoyning.

[12.] As may be seen in Fig. An. 10.

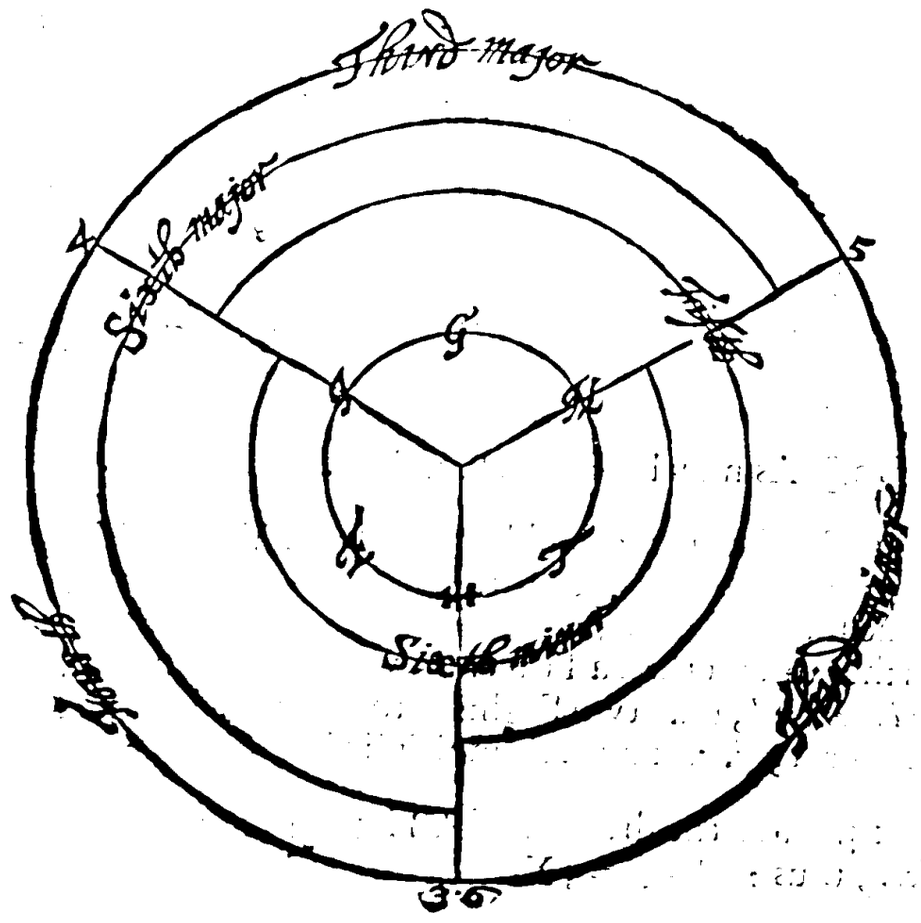
[13.] That is, the double of the lesser Term, with the greater, giveth the excesse thereof above an *Eighth*, *viz.* if the Intervall exceedeth not a *Fifteenth*: but if they be further distant than a Fifteenth, yet not exceeding a *Two and twentieth*, than two Eights is to bee added to the lesser Term; *i. e.* it must be multiplied by four: *&c.*

[14.] See the division of AB into 3: An. 10. *Arithmetically* thus: $\frac{1}{1} - \frac{1}{2} = \frac{1}{1}$ X.

[15.] *Viz.* for the graver Term. See the diviſion of A B into 4. An. 10.

[16.] For $\frac{1}{3} + \frac{1}{3} = \frac{4}{3}$.

[17.] *Viz.* p. 9. And may be made out from the diviſion of A B into ſix An. 10, if according to the method of our Authour, p. 17, wee convert one halfe thereof, *viz.* from 6 to 3 (which containeth the ſpace of an Eighth) into the Circle following; ſo that the point at 6 be joyned to the point at 3, and the Circle be divided into three equally (as is 6, 3) at 4 and 5.



[18.] As $\frac{1}{2} - \frac{4}{7} = \frac{1}{8}$ X.

[19.] Or compoſed of one, or more *Eights* only, or together with ſome one that is contained therein. p. 11.

[20.] As, in Fig. 1, An. 8, is the *Eighth* on the *Chorde* B o; *viz.* o 21 at 8.

[21.] As, on the ſame *Chorde*, is 8 21 at 14.

[22.] As, on the ſame *Chorde*, is 14 21 at 17.

[23.] It ſhould have been only the *Semitone major*; for the *Semitone minor* is not to bee found without an other Subdiviſion.

[24.] *Viz.* An *Eighth*; from the firſt diviſion of AB, p. 14: a *Fifth*; from the Second: and a *Ditone* from the Third.

[25.] 2 gives the Eight; 3 the Fifth; and 5 the Third *major*: ſee alſo A B An. 10.

[26.] Here endeth the *Former Tract*, as it's called, p. 27; l. 25.

[27.] Whereof p. 55.

[28.] By *Numbers*; as in the firſt Fig. 10. by *Diviſion*; as of the line A B, p. 14.

[29.] *Viz.* the Eighth, Fifth, and Ditone as before.

[30.] *Viz.* p. 11.

[31.] For both the compounded *Ditones*, as well as the ſimple, are to be found on a Chorde underſtood to conſiſt

ſiſt of but five equall parts; whereas the firſt compound *Fourth* requireth 8, and the Second 16; as in the Second Fig. p. 13.

[32.] Proportion is called *Multiplex*; when the greater Terme containeth the leſſer exactly twice, or oftner: *Superparticular*; when the greater containeth the leſſer once, and one certain part thereof: and *Multiplex-ſuperparticular*; when the greater doth containe the leſſer twice or oftner, and (beſides) one certain part thereof.

[33.] For, as an *Eighth*, divided equally into two parts, doth conſtitute properly a *Fifth*, and by accident a *Fourth*; ſo that *Fifth* divided into two equall parts, conſtituteth properly a *Ditone*, and by accident a *Third minor*: ſee AB Animad. 10.

[34.] For a Ditone + Fourth = Sixth *major*; a Ditone + an Eighth = Tenth *major*; and a Ditone + Fifteenth = Seventeenth *major*. See Fig. 1, p. 10, at Numbers 4 and 5; and the diviſion of AB into 5 Fig. An. 10.

[35.] For a Third *minor* + a Fourth = Sixth *minor*.

$\frac{5}{4} + \frac{3}{4} = \frac{5}{3}$.

[36.] *Viz.* of the Graver Term. See Fig. AB An. 10.

[37.] Note, that in every Muſicall *Syſteme*, (whereof there are two ſorts; the greater of Ten paralell Lines, and the leſſer of Five:) every Line is the ſeat of one Note, and every intervall of another, and therefore C is a Note higher than B, and G lower than E. See p. 40.

38. For

[38.] For $\frac{1}{8} - \frac{1}{9} = \frac{1}{16}$ *i. e.* $\frac{1}{16}$ of the Graver Term.

[39.] *Viz.* p.14, where CB, the ſpace of an Eight, is divided into CE a Ditone; ED a Third *minor*; and DB a Fourth.

[40.] *Viz.* by dividing CE p.14, equally into Two, at F: or DG, Fig. An.10. at F: or 14 21 of the Chorde B o, Fig.1, An.8, at 17.

[41.] By dividing EG, Fig. An.10, at F: or 8 14 of the Chorde B o, Fig.1, An.8, at 11.

[42.] By dividing GI, Fig. An.10, at H; or EH at G: or o 8 of the Chord B o, Fig.1, An.8, at 6.

[43.] As o 6, Fig. 1, An.8, at 2.

[44.] As DG = DE, + EF, + FG; Fig. An. 10: or 14 21, = 14 15, + 15 17, + 17 21; of the Chorde B o Fig.1, An.8.

[45.] As DE, + EF = DF; Fig. An.10: or 14 15, + 15 17, = 14 17; of the Chorde B o Fig.1, An.8.

[46.] As 14 15, with 11 14; of the Chorde B o Fig.1, An.8.

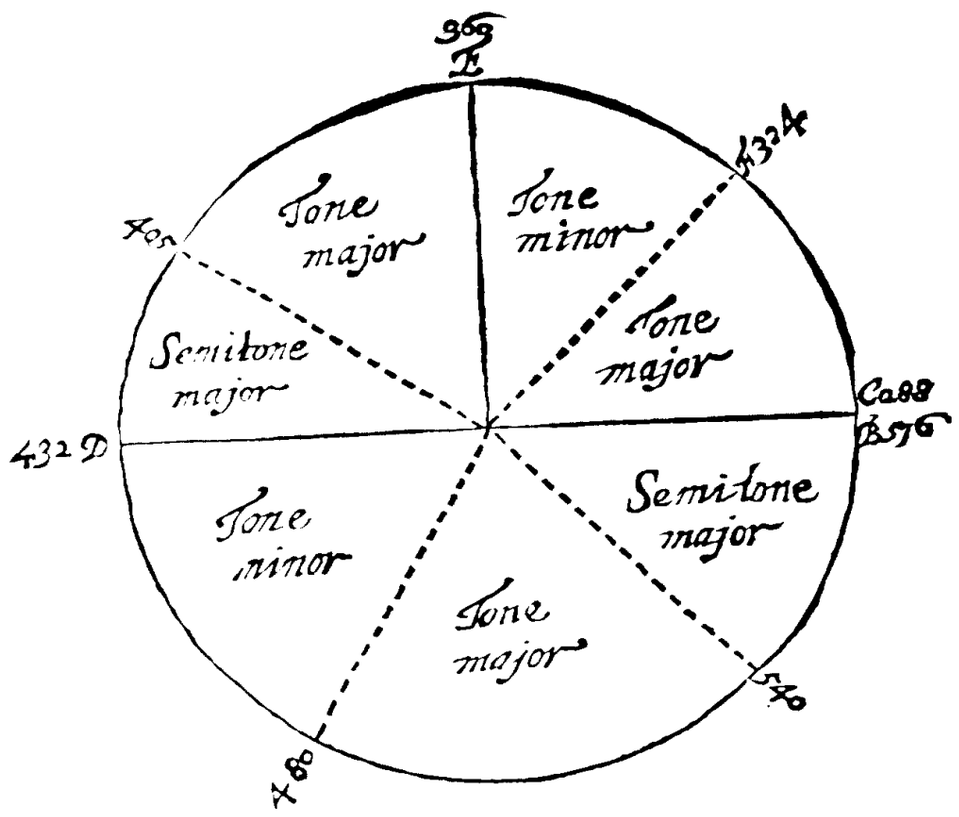
[47.] 64. 75 :: 324. 379·6875 :: 6·000. 7·031 $\frac{1}{4}$.
$24 + \frac{8}{3} = 32$. See Fig.1, An.8.

[48.] Becauſe a *Semitone majus* makes no *Conſonance* with the other two.

[49.] Becauſe a *Tone major* maketh a *Third*, with either.

[50.] *Viz.* p.27.

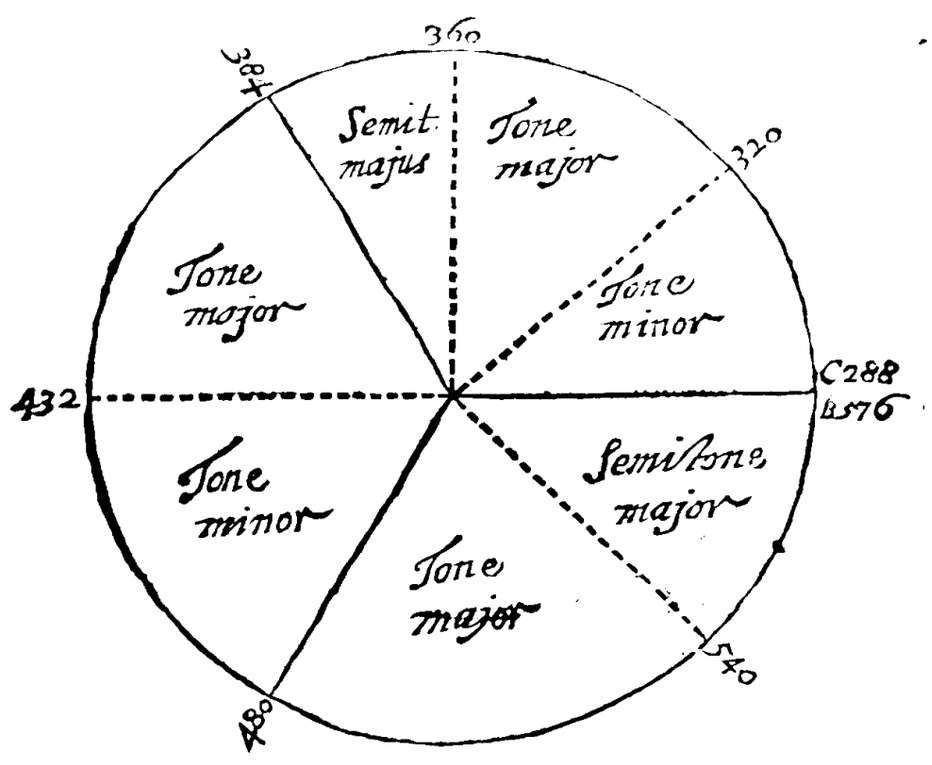
[51.] For otherwiſe a *major Semitone*, and *minor Tone* muſt fall together, as may be ſeene in this following Figure; where the ſpace of an Eighth is turned into a Circle, and divided firſt, as was CB p. 14, at D and E; and then ſubdivided as p.27.



[52.] Others do call it a *Comma majus*, See Fig. 1, An. 8.
[53.] And is called *Semitonium medium*, as Fig. 1, An. 8.
[54.] Or

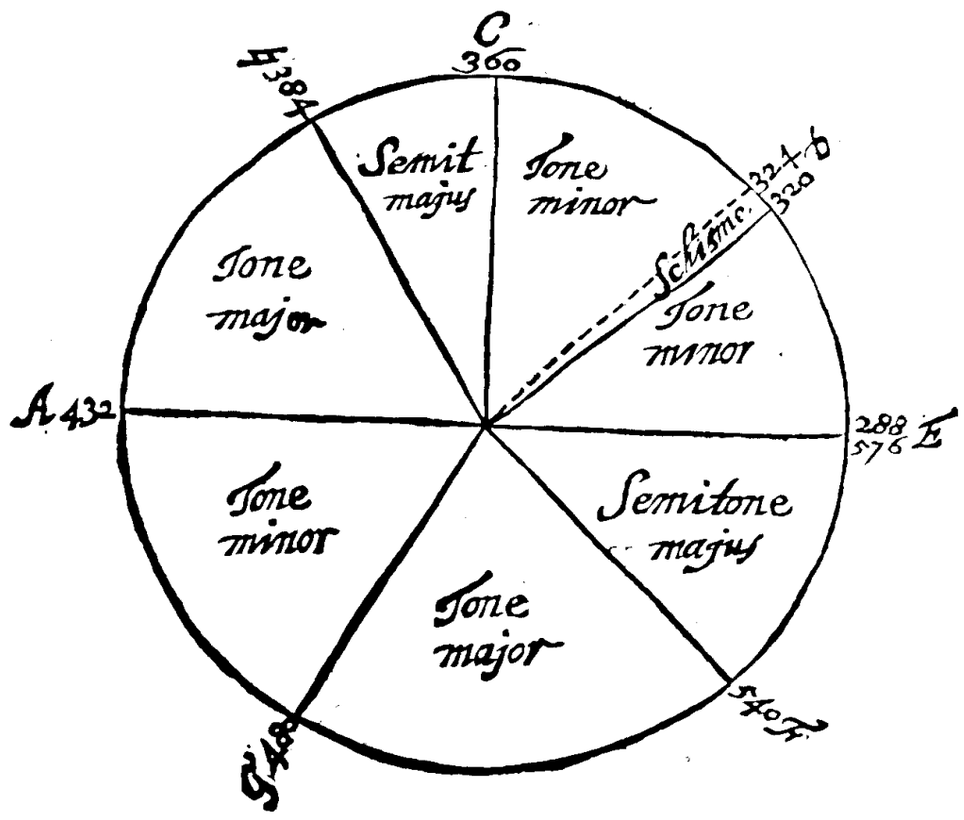
[54.] Or rather 576; becauſe it is the *Graveſt Term*, in this inſtance: as alſo according to the diviſion of an Eighth, p. 14, and 27. See Fig. An. 51.

Note that an *Eighth*, divided firſt into three equall parts, by the diviſion of the whole ſtring into ſix, as p. 13; and thoſe three then ſubdivided, as p. 28; doth give the Degrees in the ſame Order: as is to be ſeen by the following Figure, compared with the former An. 51; this only beginning a *Fourth* from the other, or the other a *Fifth* from this.



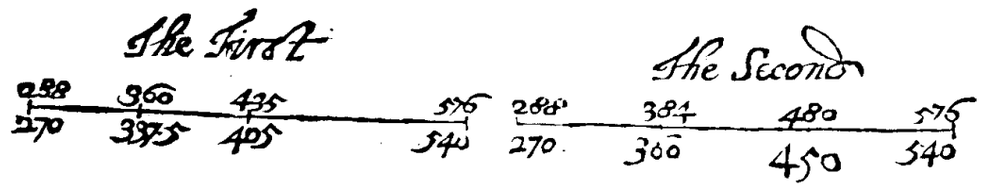
 [55.] Only

[55.] Only it ſeemeth as moved upon its *Center*, till the *Schiſme* cometh to be between 324 and 320, as this Figure doth demonſtrate; which differeth not from the laſt (An. 54): only in this the Schiſme doth ſtand divided from the *major* Tone (the Intervall between 320, and 360) in that other.



[56.] Here

[56.] Here the Authour recedeth from his former diviſion of an *Eighth*, onely by removing the *Graver Terme* from E to F: as is to bee ſeen by theſe two ſpaces of an



Eighth. The firſt divided as CB, p.14, at D and E: the Second as CI, Fig. An.10, at DG, with both which this doth accord; E, not F, being made the Graveſt Term.

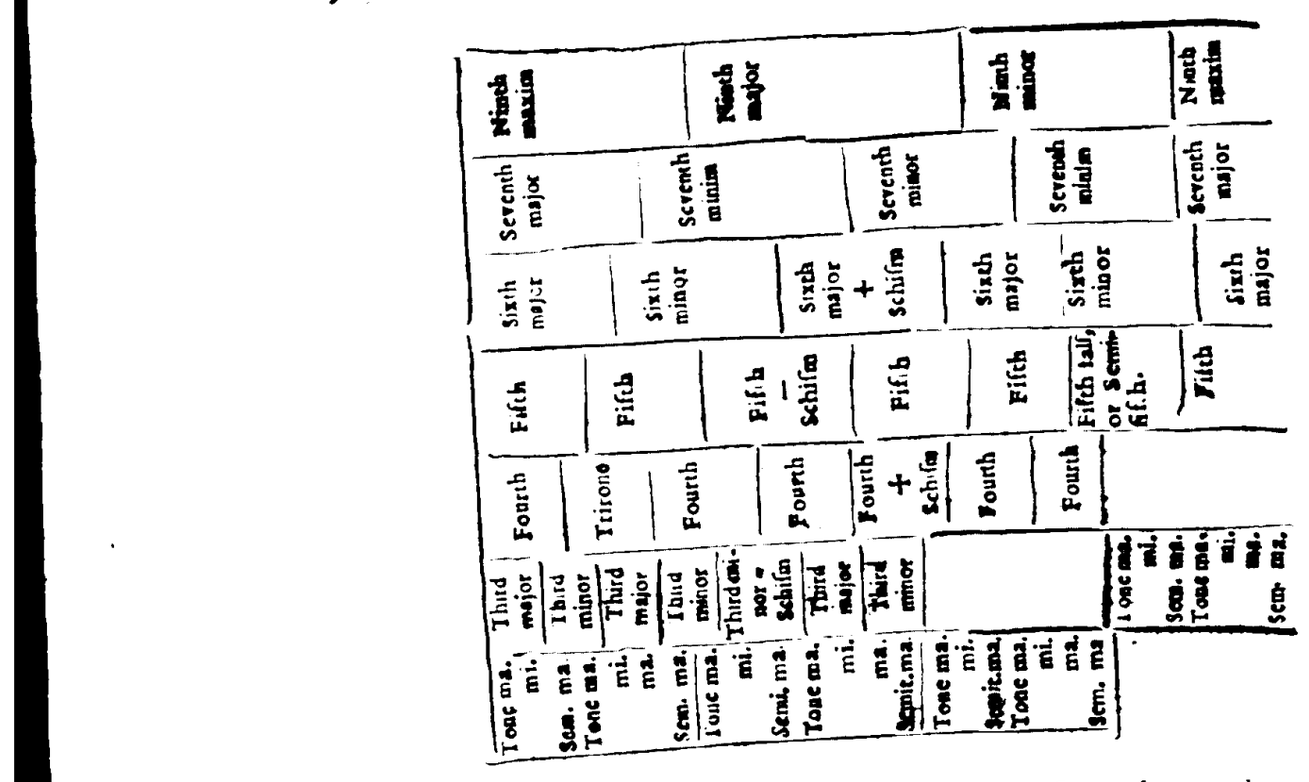
[57.] For from F(the Firſt Term of the *Voice in b flat* aſcending)to C (the firſt in the *Voice Naturall*)is a Fifth; as alſo from hence to G, where the *Voice in* ♮ *Sharp* beginneth.

[58.] For ♮ (B Sharpe) is a *Tritone* more Acute than ▽ (F being ſo accounted): and a *falſe, or Semi-Fifth* ▽ than the △. But placing the Graver Term at E; then is ♮, a Fifth more Acute than the Graver Terme; and a Fourth more Grave than the Acuter Term: and b flat a Semi-Fifth △ than ▽; and a Tritone ▽ than △. See Fig. p.35.

[59.] *Viz.* p.34. For): is F: |=| is C: and G is G.

[60.] *Viz.* Muſicall ſpaces, *i.e.* to every *Tone* the greater, and to every *Semitone* the leſſer Intervall.

[61.] As appeareth by this Figure following.



[62.] *Viz.* $\frac{128}{135}$ *Semitonium medium*, as before An. 53.

[63.] For $\frac{1}{2} + \frac{8}{9} = \frac{4}{3}$; $\frac{3}{5} + \frac{9}{10} = \frac{9}{25}$; $\frac{1}{2} + \frac{15}{16} = \frac{15}{32}$:
$\frac{3}{2} - \frac{15}{16} = \frac{8}{15}$; $\frac{3}{5} - \frac{9}{10} = \frac{5}{9}$; $\frac{1}{2} - \frac{8}{9} = \frac{9}{16}$.

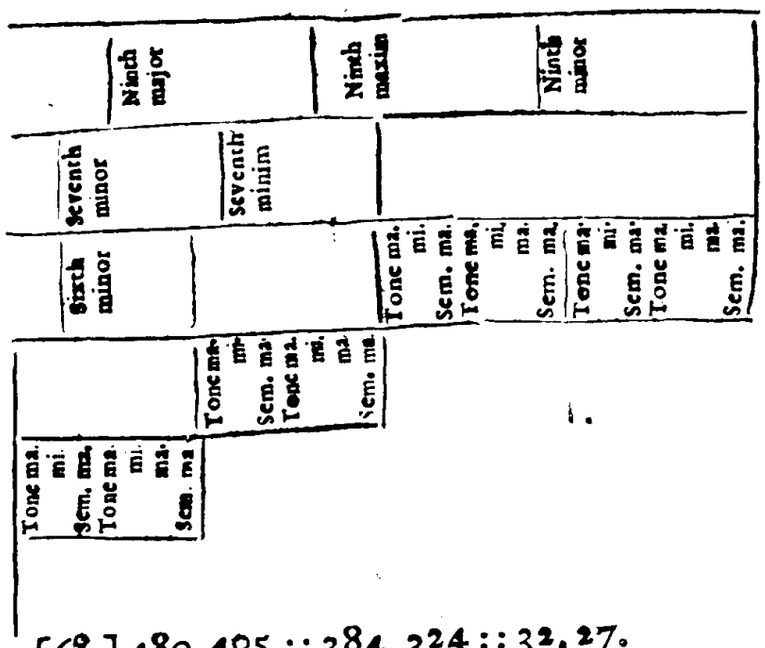
[64.] See p.22.

[65.] *Viz.* p.28.

[66.] See Figure An. 61.

[67.] For $\frac{1}{2} - \frac{30}{81} = \frac{17}{75}$; $\frac{1}{3} - \frac{40}{81} = \frac{25}{40}$: $\frac{3}{4} + \frac{80}{135} = \frac{80}{27}$; $\frac{3}{5} + \frac{16}{81} = \frac{16}{27}$.

[68.] 480.



[68.] 480. 405 :: 384. 324 :: 32. 27.
480. 324 :: 40. 27.
324. 240 :: 27. 20.
405. 240 :: 324. 192 :: 27. 16.

[69.] For $\frac{1}{\frac{3}{4}} + \frac{\frac{32}{135}}{\frac{1}{45}} = \frac{32}{45} : \frac{2}{3} - \frac{\frac{64}{135}}{\frac{1}{45}} = \frac{45}{64}$.

[70.] 540. 384 :: 405. 288 :: 45. 32.
384. 270 :: 288. 202·5 :: 576. 405 :: 64. 45.

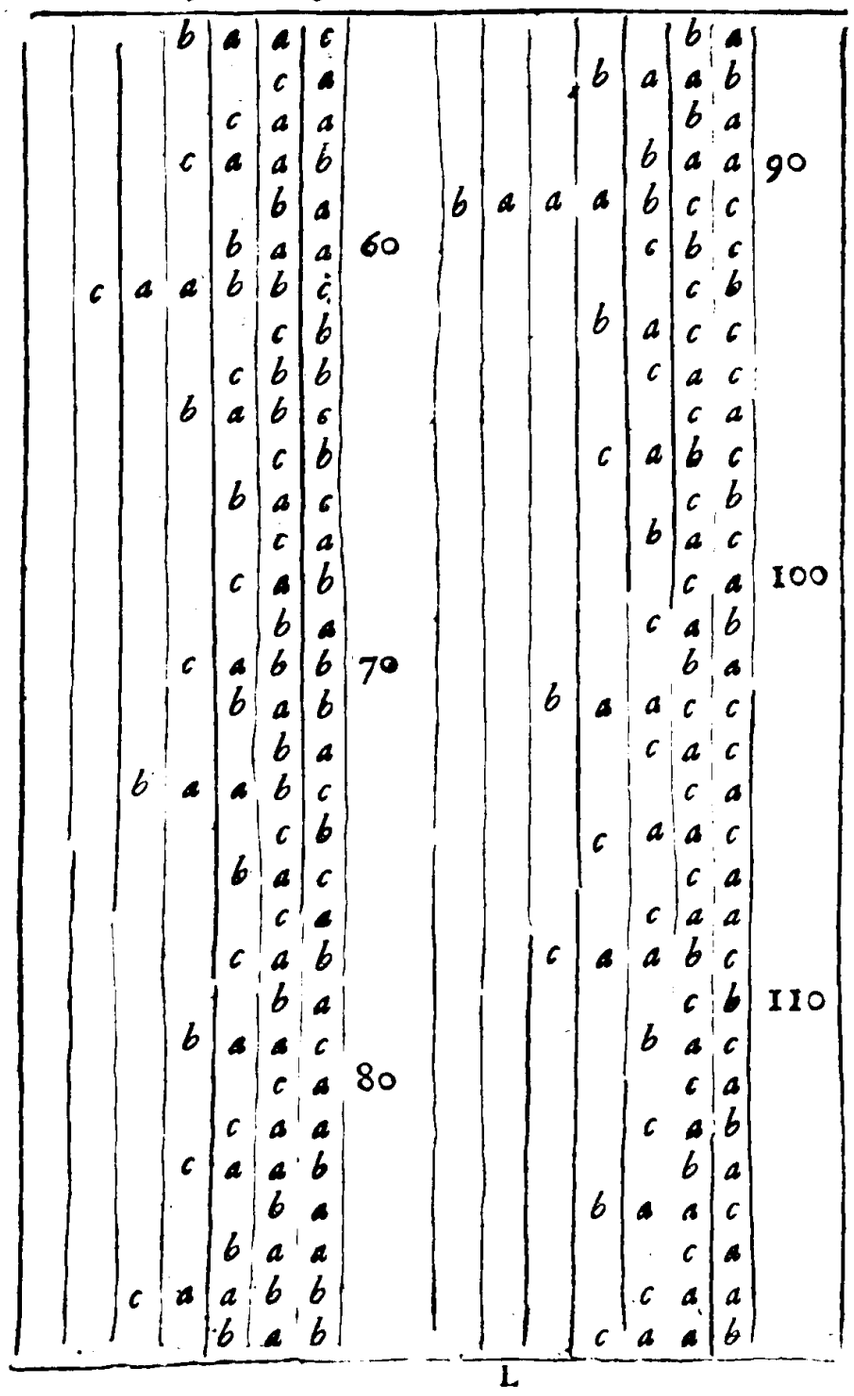
[71.] *viz.* the first compound *Eighth*, *i. e.* a *Fifteenth*.

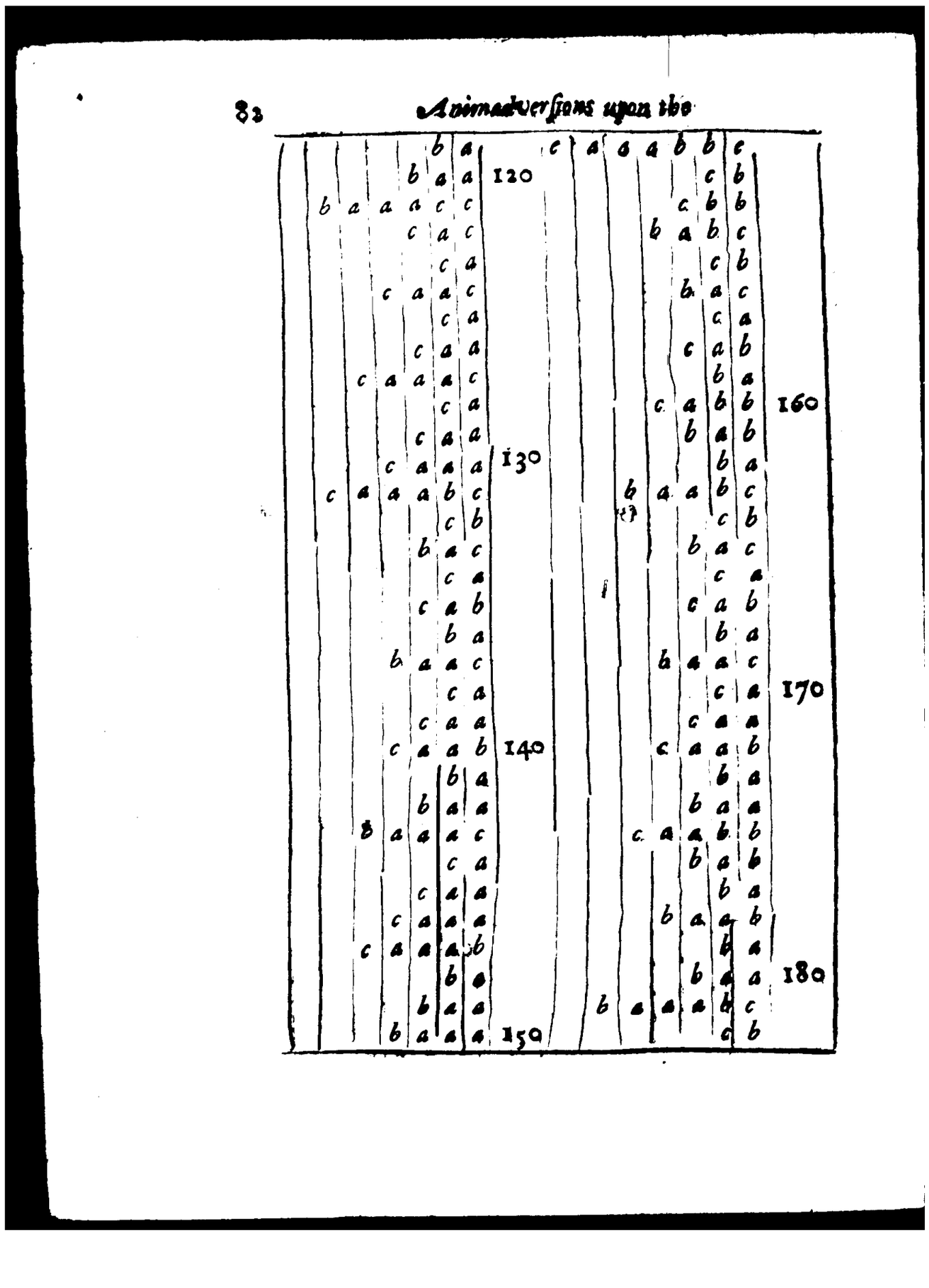
[72.] *Viz.* without altering the order of Succession, p. 30, and 41.

Otherwise, of Eighths considered only as consisting of three *major* Tones, two *minor* Tones, and two *major* Semitones;

mitones ; there are 210 ſeverall ſorts, or *Moods* ; and may be found, by the Laws of *Combination*, as in this *Table* following ; where note *a* is put for a *major Tone* ; *b* for a *minor Tone*, and *c* for a *major Semitone*.

a	a	a	b	b	c	c					c	a	b	b	
				c	b	c						b	a	b	
					c	b							b	a	30
			c	b	b	c			b	a	a	b	c	c	
					c	b						c	b	c	
				c	b	b							c	b	
		b	a	b	c	c					b	a	c	c	
				c	b	c						c	a	c	
					c	b							c	a	
			b	a	c	c	10				c	a	b	c	
				c	a	c						c	a	b	
					c	a						b	a	c	
			c	a	b	c							c	a	40
					c	b						c	a	b	
				b	a	c							b	a	
					c	a				b	a	a	c	c	
				c	a	b						c	a	c	
					b	a							c	a	
		c	a	b	b	c					c	a	a	c	
					c	b	20						c	a	
				c	b	b						c	a	a	
			b	a	b	c				c	a	a	b	c	
					c	b							c	b	50
				b	a	c						b	a	c	
					c	a							c	a	
				c	a	b						c	a	b	
					b	a							b	a	


60
70
80
90
100
110



		b	*a*	*c*			*c*	*a*	*a*	*a*	*b*	
			c	*a*						*b*	*a*	
		c	*a*	*b*					*b*	*a*	*a*	
			b	*a*				*b*	*a*	*a*	*a*	200
	b	*a*	*a*	*c*		*c*	*a*	*a*	*a*	*b*	*b*	
			c	*a*					*b*	*a*	*b*	
		c	*a*	*a*						*b*	*a*	
	c	*a*	*a*	*b*	190			*b*	*a*	*a*	*b*	
			b	*a*						*b*	*a*	
		b	*a*	*a*					*b*	*a*	*a*	
b	*a*	*a*	*a*	*c*			*b*	*a*	*a*	*a*	*b*	
			c	*a*						*b*	*a*	
		c	*a*	*a*					*b*	*a*	*a*	
	c	*a*	*a*	*a*				*b*	*a*	*a*	*a*	210

After the same *Method*, there are found twelve *Fifths*, and six *Fourths*, as followeth.

Fifths

a	*a*	*b*	*c*	1	*b*	*a*	*a*	*c*	7
		c	*b*				*c*	*a*	
	b	*a*	*c*			*c*	*a*	*a*	
		c	*a*		*c*	*a*	*a*	*b*	
	c	*a*	*b*				*b*	*a*	
		b	*a*	6		*b*	*a*	*a*	12

Fourths

a	*b*	*c*	1
	c	*b*	2
b	*a*	*c*	3
	c	*a*	4
c	*a*	*b*	5
	b	*a*	6

And therefore of *Eighths* divided into *Fourths* and *Fifths*, there are seventy and two severall *Moods*: and thus of *Fifths*, divided into *Thirds*, there are eight *Species*: *&c.*

[73.] *Viz.* both *Arithmetically*, as 2 3 4, the Fifth before

fore the Fourth ; and *Harmonically*, as 3 4 6, the Fourth before the Fifth, aſcending.

[74.] *Viz.* from *B* to *B*, *Arithmetically*; and from *E* to *E*, *Harmonically*, in *b* flat : or from *F* to *F*, *Arithmetically* ; and from *B* to *B*, *Harmonically*, in ♮ (*B* ſharp) p. 41.

[75.] *Viz.* from *E* to *E*, in *b* flat; or from *B* to *B*, in ♮. p. 41.

[76.] *Viz.* from *F* to *F*, *A* to *AB* to *B*, and *E* to *E*, in *b* flat ; or from *C* to *C*, *E* to *E*, *F* to *F*, and *B* to *B*, in ♮. p. 41.

[77.] *Viz.* the two Extreams, and the midle Term.

7 8.] See p. 18 and 30.

§ 1. Now conſidering (as was ſayd An. 1 and 3) that not the *viſible* proportion of *Chords* or *Strings*, but the *audible* proportion of their *Sounds* only is conſiderable in Muſick ; and that, by the *Sence* of *Hearing*, wee doe judge of *Sounds* according to the *Geometricall*, not Arithmeticall Proportion, or proportionall *Diviſion* of the *Strings*, that give them : I conceive it was rightly inferred An. 3, that *Chordes*, as to Sounds, ought to bee divided according to a *Geometricall*, not Arithmeticall *Progreſſion* ; by force of the ſame *Reaſon* (adequated to the Sence of Hearing) which our *Authour* gave for the contrary opinion in his ſixth Preconſiderable. It therefore remaineth that I heere ſhew what *Diviſion* it is I mean, and how it may be performed.

§ 2. Firſt then let the Chord *AZ*, Fig. 2, An. 8, be divided at *S*, into *Extream and Mean Ration*; by 30. 6. *Elem. Euclid.* or by *Prob.* 1, *c.* 19, *Clavis Mathematicæ*; which done, let AS, the *Mean Proportionall*, bee divided into 17 *equall Semitones*, by 16 mean Proportionals; by the Latter Table of

of *Poteſtates* Chap. 12. of Mr. *Oughtreds Clavis Mathem.* or rather (the other way, in this caſe, being very laborious) *Chap.* 17. *Arithmeticæ Logarithmicæ H. Briggij.*

§ 3. *I perform'd it thus.*

AZ = B
AS = A
Therefore ZS = B - A
B — A. A :: A. B.
Aq = Bq - BA
Aq + BA = Bq
Aq + BA + ¼Bq = Bq + ¼Bq
A + ½B = √: Bq + ¼Bq:
A = √: Bq + ¼B: - ½B

B = 10
Bq = 100
¼Bq = 25
Bq + ¼Bq = 125
√: Bq + ¼Bq: = 11·18033, 98875 -
½B = 5
A = 6·18033, 98875 -
B - A = 3·81966, 01125 +

B = 10·00000, 00000 Z 1, 00000, 00000
B - A = 3·81966, 01125 Z 0, 58202, 47162
X 0, 41797, 52838
17
0, 02458, 678.4 = R 1.058 +

Z B - A 0, 58202, 47162 = ZS 3·820 -
+ ZS 0, 60661, 14976 ZR 4·042 +
ZR 0, 63119 82790 ZQ 4·178 -
ZQ 0, 65578, 50604 ZP 4·327 -
ZP 0, 68037, 18418 ZO 4·790 +
+ ZO

ϛ + ZO	0, 70495, 86232	=	ZN	5·069 +
ZN	0, 72954, 54046		ZM	5·365 −
ZM	0, 75413, 21860		ZL	5·677 +
ZL	0, 77871, 89674		ZK	6·008 −
ZK	0, 80330, 57488		ZI	6·358 −
ZI	0, 82789, 25302		ZH	6·728 +
ZH	0, 85247, 93116		ZG	7·120 −
ZG	0, 87706, 60930		ZF	7·535 −
ZF	0, 90165, 28744		ZE	7·974 −
ZE	0, 92623, 96558		ZD	8·438 +
ZD	0, 95082, 64372		ZC	8·929 +
ZC	0, 97541, 32186		ZB	9·450 −
ZB	1, 00000, 00000		ZA	10·000

§ 4. Into *Extreame and meane Ration*; that the parts and whole may be ∺. ZS. SA : : SA. ZA.

§ 5. Into *Seventeen equall Semitones*; because (the Ear not well distinguishing smaller Intervalls) this *Number* doth best admit of the subsequent *Divisions*, proportionall to their *Extreames*; whence the *Consonances* doe naturally arise, according to this *Analogy*, *viz.* As the number of parts in the First Terme, is to the number of parts in the Third; so the number of Rations between the First and Second, to the number of Rations between the Second and Third. And may bee work'd by either of the following Rules.

In Naturall *Numbers.*

First Rule. △ ~ √ [△/▽] ♃ [△] ℞ = Second Terme.

Second Rule. ▽ ϛ √ [△/▽] ♃ [▽] ℞ = Second Terme.

In

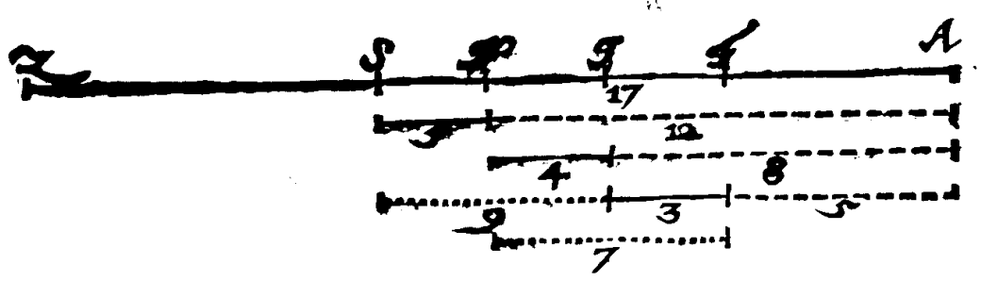
In Artificiall *Numbers, or Logarithmes.*

First Rule. $b + \frac{aB - bB}{A + B} =$ Second Terme.

Second Rule. $a - \frac{aA - bA}{A + B} =$ Second Terme.

Note $\frac{\triangle}{\nabla} = \overline{\nabla} + \underline{\triangle} : A = \overline{\nabla} : a = \frac{z}{a}\,\overline{\nabla} : B = \underline{\triangle} : b = \frac{z}{a}\,\underline{\triangle}$.

§ 6. For, from this Division, of the Intervall of an *Eleventh* (*i.e.* the Meane Proportionall AS); ariseth an Eighth, and a Fourth: of an *Eighth*; a Sixth *minor*, and a Third *major*: and of a *Sixth minor*; a Third *minor*, and a Fourth, and these compounded give the rest.



ZS. ZA :: 5 Semit. 12. *fere.*
ZN. ZA :: 4. 8. *fere.*
ZI. ZA :: 3. 5. *fere.*
Third *minor* = 3 Semitones.

Third

Third *major* = 4 Semitones.
Fourth = 5.
Fifth = 7.
Sixth *minor* = 8.
Sixth *major* = 9.
Eighth = 12.

This Proportion or Progreſſion, from its excellency and compoſion, I call Ratio-harmonicall.

§ 7. It may bee objected that the R of ZS to ZA is 2·6180339887 –, that is as 5 to 13 +; and therefore SA ought rather to have been divided into 18 proportionall parts, by 17 Meane Proportionalls: whereof 5 = Intervall of a Fourth; and 13 = Space of an Eighth.

§ 8. To which I anſwer, that SA is underſtood to bee divided into 13·819660 1125 + Proportionall parts: (becauſe the R of ZS to ZA, *viz.* 2·6180339887 5 –, is as 3·81966,01125 + to 10·00000, 00000.) whereof the ſpace of an *Eighth* containeth 10·00000, 00000; and of a *Fourth* 3·81966, 01125 +. *&c.* And may bee eaſily found (by Logarithmes) working, according to the Second Rule, Par. Fifth, thus.

AZ = 10·00000,00000 ℔ 1,00000,00000.
ZS = 3·81966,01125 ℔ 0,58202,47162.
0,41797,52838,00000000000.
13·819660 1125
0,30244 97566.
0,69755,02434· = ZN, 4·98368,11082.

AZ = 10·00000,00000 ℔ 1,00000 00000.
ZN = 4·98368,11082 ℔ 0,69755,02434.
0,30244,97566,00000000000.
14·98368 11082
0,20185,27720.
0,79814 72280· = ZI, 6·28271,31146

ZA

ZA = 10·00000,00000 ∛ 1,00000,00000.
ZI = 6·28271,31146 ∛ 0,79814,72280.
0,20185,27720,00000000000.
16·28271,31146
0,12396,75296.
0,87603,24704. = ZF. 7·51679,09301

§ 9. But this exactneſſe is not requiſite, ſince the *Senſe* of *Hearing* is not ſo perfect, as to confine the *Conſonances* to ſo preciſe a *Meaſure*; (ſee p. 46.) and therefore, ſeeing that SA divided into 17 Proportionall Spaces, doth give (without any *Fraction*, or ſenſible difference,) all the ſimple *Conſonances*; & that $\frac{38\cdot1966}{100\cdot0000} = \frac{47745}{12\cdot5000}$ that is, without Fraction, $\frac{4}{12}$; as becauſe, if SA be divided into 18 Proportionall *Intervalls*, NA (containing 13 of them) cannot bee divided at I without a *Fraction*, much leſſe again at F, I made 17 ⊂ Par. 3. with which the common *Diviſion* doth not ill accord; for ſo many *Semitones* are contained in an *Eleventh*.

§ 10. Thus then having reſolved that the *Proportion* of ZS to ZA is, as to the practick, exactly enough accounted as 5 to 12: It muſt follow, by force of the preceding Rules Par. 5. that (1) the *Product* of 38·1966, 01125 *Multiplyed* by the Seventeenth *Roote* of the Fifth *Poteſtas* of 2·61803398875; or (2) the *Quotient* of 10·00000, 00000 *Divided* by the Seventeenth *Roote* of the Twelfth *Poteſtas*, of 2·61803398875 = ZN. And by *Logarithmes* as followeth.

AZ = 10·00000,00000	♃	1,00000,00000	
ZS = 3·81966,01125	♃	0,58202,47162	
	X	0,41797,52838	0,41797,52838
	♂	5	12
	♄	2 08987.64190	5,01570,34056
	⊏	17	17
	⊑	0,12293,39070	0,29504,13768
	♃△	0,58202,47162	▽ 1,00000,00000
	Z	0,70495,86232	X 0,70495,86232

the *Logarithme* of (ZN) 5·0694. differing from the former, Par. 8, about the *Interval* of a *Schisme*, or *Comma*, *majus*, no preceptible *Dissonance*, as p. 33.

§ 11. Then ZN being to ZA, as 1 to 2 *fere*; therefore, by the Second Rule in Logarithmes, Par. 5.

♃ ZA	1,00000,00000
ZN	0,70495,86232
	0,29504,13768
	2
	0,59008,27536
	3
	0,19669,42512
	1,00000,00000
	0,80330,57488 ♃ ZI, 6·358

§ 12. Lastly ZI and ZA being as 3 to 5 *fere*; therefore

♃ ZA	1,00000,00000
ZI	0,80330,57488
	0,19669,42512
	5
	0,98347

0,98347,12560
8
0,12293,39070
1,00000,00000
0,87706,60930 ℥ ZF, 7535-

§ 13. *With what hath been here ſaid, if the Reader pleaſe to be ſatisfied at preſent; I ſhall, when, if ever, I have* (God mercifully aſſiſting) *laboured through my tedious Troubles and Diſtractions, endeavour his better compenſation with an entire and particular* Tract, *according to this new* Theory. (*And hence too ſhall ſhew how* Aſtrologers *may deduce their* Aſpects; *with more, I preſume, of ſatisfaction, than from any other hitherto diſcovered to them. And perhaps with ſomewhat elſe more worthy the Reader's paines, and mine.*) *If not; I here further preſent him the two following* Diviſions *of a* Chord, *and will ſo leave him to ſeeke it there, or where elſe he pleaſeth.*

§ 14. The *One* (approved by many Excellent Mathematicians; See *Merſennus Lib.* I. *de Inſtrumentis Harmonicis, Prop.* 15.) is the *Diviſion* of ZA, Fig. 3, An. 8, firſt into two equall parts at N; and then of NA into twelve equall *Semitones*, by eleven *Meane Proportionalls*, according to this *Table* following.

	In Species,	*Numbers Surde,*
ZN	E = △. ZN.	5 000
ZM	√ 12 AEcccq.	√cccc 488281250.000000000,000 00000.000,00000000
ZL	√ 6 AEcq.	√cc 31250 000000,000000,000000
ZK	√ 4 AEc.	√qq 1250.0000,0000,0000
ZI	√ 3 AEq.	√c 250.000,000,000
ZH	√ 12 AcqEcqq.	√cccc 781250000.000000000000,000 00000000,0000000000
ZG	√ AE.	√ 50.00 00,00
ZF	√ 12 AcqqEcq.	√cccc 3125000000.0000000000,00 00000000,00000.00000
ZE	√ 3 AqE.	√c 50.000,000,000
ZD	√ 4 AcE.	√qq 500.0000,0000,0000
ZC	√ 6 AcqE.	√cc 50000.000000,000000,000000
ZB	√ 12 AcccqE.	√cccc 5000000000.0000000000,0 00000000,00000000
ZA	A = ▽. ZA.	10.000

	Logarithmes,	*Numb. D.*
ZN	0, 69897, 00043.	5'000
ZM	0, 72405, 58372.3	5'297+
ZL	0, 74914, 16702.2	5'632+
ZK	0, 77422, 75032.1	5'946+
ZI	0, 79931, 33362.	6'300−
ZH	0, 82439, 91691.3	6'674+
ZG	0, 84948, 50021.2	7'071+
ZF	0, 87457, 08351.1	7'492−
ZE	0, 89965, 66681.	7'937+
ZD	0, 92474, 25010.3	8'409−
ZC	0, 94982, 83340.2	8'909−
ZB	0, 97491, 41670.1	9'439−
ZA	1, 00000, 00000.	10'000

§ 15. Tim

§ 15. The *other* is the Division of ZA, Fig. 4. An. 8, *Harmonical* or Q: and of QA into 15 equall *Semitones*.

The manner thus.

ZA = B
ZQ = A
Therefore QA = B − A

A.B : : B − 2A.A.
Aq = Bq − 2BA
Bq = Aq + 2BA
2Bq = Aq + 2BA + Bq
√:2B: = A + B
√:2Bq: − B = A

B = 10.
Bq = 100.
2Bq = 200.
√:2Bq: = 14·1421+
A = 4·1421+
B−A = 5·8579−

B = 10.000 ∴ 1, 00000, 00.
A = 4·142+ ∴ 0, 61728, 48.
X 0, 38277, 52.
⌐ 15:
⌐ 0, 025511, 83 = R 1·067−
∴A 0, 61728, 48. = ZQ 4·142+
⌐ +ZQ 0, 64274, 31·7 ZP 4·393−
ZP 0, 66826, 14.14 ZO 4·659−
ZO 0, 69377, 98.6 ZN 4·941−
ZN 0, 71929, 81.13 ZM 5·240−
ZM 0, 74481, 65.5 ZL 5·557−
ZL 0, 77033, 48.12 ZK 5·893−
ZK 0, 79585, 32.4 ZI 6·250−
ZI 0, 82137, 15.11 ZH 6·628−
ZH 0, 84688, 99.3 ZG 7·029−
ZG 0, 87240, 82.10 ZF 7·454+
ZF 0, 89792, 66.2 ZE 7·905+
ZE 0, 92344, 49.9 ZD 8·384−

 ⌐+ ZD

+ ZD 0,94896,33.1 ZC 8·891 +
ZC 0,97448,16,8 ZB 9·429 +
ZB 1,00000,00. ZA 10·000

§ 16. And lastly, that the *Reader* may, with the lesse trouble, compare these severall *Divisions* each with other; I have both reduced our *Authours* Numbers to these, and these to his. See Fig. 1, 2, 3, and 4. An. 8.

FINIS.

P.	L.	*These Errors*	*Amend thus.*
3	16	,[5]	[5],
7	24	Conſonancies	Conſonances
	25		
	26		
8	6		
11	25		
	26		
12	1		
	24	Hexarhordon minus,	Hexachordon minus,
17		Eight	Eighth
	2	Diapaſſon	Diapaſon
19	23	an Eighth [26].	[26] an Eighth [26].
18	30	o Fi dne	a Fifth, and
21	1	deſumded	deſumed
22	6	For Example,	than, for Example, betweene
23	6	than betweene	
	7		
25	10	Muſicions	Muſicians
34	8	Muſitians	
44	13	9/5	5/9
	30	obſerved, that a voyce [65] doth	obſerved [65], that a voice doth
53	19	Syncoap	Syncopa.
54	30	a Eighth,	an Eighth.
55	3	a Vniſon.	an Vniſon.
65	22	* ▽	▽
	23	△	△
69	18	* Eights	Eighths
71	2		
	14	Eight	Eighth.
73	2		Superior Tractate,
72	26	Former Tract,	Fig. p. 10.
	19	Fig. 10	10; and thoſe three then ſubdivided,
75	6	133; and thoſe three then ſubdivided, as p. 283	as p. 27.
78	7	5/3	2/3
84	8	A to AB to B,	A to A, B to B,
85	3	Chap.	Cap.